A Dollar Weighted Utility Model for Purchasing Insurance Policy

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Abstract
The purpose of this paper is to suggest a new method for presenting options to insurance customers in order to provide clarity in the process of evaluating risky decisions when purchasing insurance. A point of interest for the insurance industry is how customers consider the fairness of an insurance contract based on the probability of a claim and the certainty equivalent of paying a fixed amount up-front to mitigate uncertain losses. Insurance providers are better able to meet the needs of their customers when they are able to communicate the utility their policies provide. This paper extends formulas currently presented as prospect theory into a format available to the insurance industry in order to improve customer awareness of the value of any given insurance policy. The goal is to identify a single score that, presented in dollar amount, would offer equivalent utility to a prospective customer regarding the risk being mitigated. A targeted advantage of the method of arriving at a single payout utility equivalent (SPUE) is that the approach can be generalized to apply to most types of insurance as well as other types of decisions based on risky outcomes, such as lotteries or gambles.
1. Introduction

When an insurance company sells its products, it provides its buyers with a sense of confidence that the insurance contract offers value. Insurance companies also gain business by providing new customers assurance that paying their premium will provide the customer with some economic good. For contracts that never materialize in an insurance claim, the value provided is a peace of mind for the insured that a specific risk has been mitigated. That peace of mind, experienced as a sensibility about having the insurance contract, is not something insurance agents are used to quantifying. During the insurance buying process, the buyer is currently presented with proposed coverage amounts and corresponding premium rates, but the underlying probabilities on which the insurance prices are based are typically unavailable to the buyer and often even to the agent.

In order to provide the insurance buyer with more information about the value of the contract to be purchased, this paper proposes an additional item of information to present to the buyer during the decision making process. The goal is to identify a single score that, presented as a dollar amount, would offer equivalent utility to a prospective customer for the risk being mitigated. A targeted advantage of the method of arriving at a single payout utility equivalent (SPUE) is that the approach can be generalized to use in most types of insurance as well as other types of decisions based on risky outcomes, such as lotteries or gambles.

A broad range of applications is appealing in that it allows for a common index to build around the concept in multiple contexts, improving its accessibility similar to the consumer credit scores. Also, by following the formulas already developed by Kahneman and Tversky (1979, 1992) to predict human decision making in the realm of risky outcomes, the approach to calculating a SPUE for a specific insurance policy provides a unique and objectively identifiable solution for any given set of probabilities, payouts, and risk preference parameters. This model also aligns with an accepted theory on risk aversion and loss aversion as observed in decision making based on risky outcomes.

A. Confusion for the Insurance Customer

Insurance buyers typically face a challenge in reaching the conclusion that they are receiving fair value for the dollars of premium paid for insurance coverage. Questions arise as to what constitutes a fair value for mitigating liabilities whose payments are uncertain. Insurance customers can also experience confusion and frustration in the process of buying an insurance contract and maintain a level of skepticism about the insurance companies as the counterparty (Brown, 1977).

Customers purchasing insurance lack clarity on the many provisions that are standard for the contract forms that constitute the insurance agreement. When referring to the insurance buying process, Abraham notes, “[M]ost individuals know little about what they are
purchasing… For virtually all individuals, insurance policies are complex documents with terms they neither read nor understand” (2013, p. 660). Consumers can feel daunted by the task of identifying the appropriate level of coverage and gauging the value of insurance products offered at a given rate.

In order to neutralize the asymmetries inherent in the complicated insurance pricing process, which structurally favor insurers, premium rates traditionally require regulatory approval (Abraham, 2013). Measures are in place to protect the consumer from unfair practices by the insurer stemming from the information asymmetry. But at present no universal and intuitively understood signal exists for insurance that would allow for a comparison by the consumer to capture the value of the experience of having a risk covered.

The insurance premium offers a signal of market clearing price for a given type of insurance, but the price of a policy does not necessarily convey to the consumer a measure of the probability of the underlying event. Insurance prices are built from more information than just risk, including loading to cover expenses and profit as well as downward pressure from competition.

Further complicating the matter is the question of utility. Economic utility is the capacity of a good or service to satisfy the want of a consumer. The insurance buyer can benefit from knowing a measure of the utility provided by a given policy, besides the price, which may be intuitively compared to similar insurance policies or even other types of insurance.

Unlike material consumables or other experience goods, customers of insurance do not possess anything tangible upon the purchase of an insurance contract. Claims offer an opportunity for the policyholder to realize the value of an insurance contract at the instance of an unfortunate event, but for most policyholders, the nature of the contract is designed in such way that a claim and its corresponding payout are unlikely events, such as an automobile accident or a weather event. A challenge exists for insurers to identify and communicate the value that will be subjectively perceived as economic utility for their customers.

The desire to understand the utility being purchased in an insurance contract goes beyond understanding the face amount covered by the contract. The insurance buyer needs some measure of the underlying probabilities that dictate the nature of the risk to be insured. The fact that the insurer possesses the advantaged position in an informational asymmetry constitutes a market failure. Here, the disadvantaged party, i.e. the insurance buyer, may not be able to gauge the fair value of an insurance contract that is offered. Lacking insight into the fair value, the buyer lacks a means of comparing policies with different features or coverage amounts. The buyer is forced to rely on the reputation of the insurer and the attitudes of other buyers to confirm that the insurance policy will fulfill the buyer’s needs.
Even the concept of fair value comes into question as the buyer is unaware of the specific calculations used to price the product. Insurance companies possess experience tables which offer an ability to predict how a given block of insured policies claims history will emerge over time. While it is acknowledged that individual policyholders may have better information than their insurers in the case of some specific risks (leading to adverse selection), insurers usually possess superior information when it comes to general risk (Seog 2009).

With the knowledge of past experience and predictions about future claims, the insurer has the ability to calculate how much of an insurance policy’s premium is allotted for claims and how much is designated as loading for expenses and profit. What the insurance buyer currently lacks is some representation of the measure of the underlying probabilities that go into the calculation of premium, as well as some recognition of the economic utility associated with the payouts.

This paper develops a method for arriving at such a score, which is denominated in dollars. The advantage of providing a single score, in terms of dollars, is that the consumer readily has a grasp of how dollars translate into utility. Except here, the dollars, instead of communicating the price, represent a value of money the utility models predict the customer would be indifferent toward either losing or facing the risk of loss unmitigated by insurance. The dollar amount represents the trade-off.

In the case of a positive payout, the dollar amount conveys how much a decision maker would be willing to accept to forgo an uncertain (and hence risky) option to earn a potentially higher amount, if the probabilities of obtaining such amount are established beforehand. In the case of a negative payout, the dollar amount conveys how much a decision maker would be willing to pay to forgo an uncertain option to avoid losing a potentially more unfavorable amount, again if the probabilities are established beforehand. An important aspect of the single score under discussion is that it is based on a principle as if the underlying probabilities are identified a priori.

A relevant aspect when discussing how insurance companies present their products to consumers is to identify what motivates the buyer to enter the insurance market. The reasons for buying insurance can help identify how motivated customers will be able to shop around for competitive prices and how willing they are to forgo insurance if the price is higher than expected. For some types of insurance, coverage is mandated, either as part of a financial contract such as a mortgage or loan, or required by the state as a prerequisite to a license or freedom. For insurance that is mandated, the provider does not necessarily find a need to justify the utility of its product in order to make sales to the public. Here brand recognition, convenience, and competitive price play the major parts in deciding if the customer will buy.
For other types of insurance, such as product warranties or voluntary life insurance, where the customer has some option not to purchase coverage, the insurer must be capable of convincing customers that purchasing an insurance contract provides some economic good in exchange for the premiums paid. Presumably, when insurance is opted for, the utility gained from knowing their risk is mitigated compensates the insurance customer for the premium forgone up front.

Given that consumer sentiment affects sales of insurance in the voluntary market, insurance companies will find advantage in assessing the subjective experience of engaging in the insurance buying process. For the voluntary market, a single score that conveys the utility of the policy can serve as a selling feature. However, even when insurance is mandatory, conveying the utility of the policy about to be purchased may counter some of the unpleasantness of being required to buy insurance.

The problem with establishing the value of the insurance product purchased goes beyond the structural nature of the insurance transaction. Our hypothetical buyer looks for a normative rule he can follow that will tell him how much insurance he should buy at a given rate. Our buyer looks for a measure of whether or not his insurance purchase is rational.

It is useful, in order to build an understanding of how people come to make decisions about insurance, to review the approaches that have been taken to generalize how risky prospects are rated with respect to utility. After all, the decision to purchase insurance is just one way in which people consider between prospects with probabilistic outcomes. The following section will describe the expected utility theory which has served as both a descriptive and normative model of how decisions are made in a risky context.

B. The Expected Utility Theory

One model which has evolved to describe how people make decisions on risky outcomes is the expected utility theory. The theory predicts that the certainty equivalent of the utility gained (or lost) from risky decisions which entail more than one possible outcome is equal to the sum of the utility of each outcome weighted by the probability of that given outcome. In order to enable the prediction of human behavior when faced with choices involving risk, Friedman and Savage propose the Expected Utility Hypothesis as a descriptive model (Friedman & Savage, 1952).

Expected utility differs from the actuarial value in that the utility function is concave in the former. Actuarial value considers value of the payouts in each prospect, the time value of the money exchanged, and the probability of obtaining each outcome. It does not consider the change in preference of the individual for increasing payouts.

The Expected Utility Hypothesis predicts that a rational person will display risk aversion (a preference for certain gains over higher expected utility found in riskier outcomes) when selecting from two or more risky choices (Friedman & Savage, 1952). The development of the
concave utility function came about through considering a problem called the St. Petersburg paradox.

The hypothetical game begins with the winning stake set at 2. A coin is flipped. Each time the coin lands on heads, the winning stake is doubled. When the coin lands on tails, the game ends and the stake is paid out. The number of coin flips is not limited at the beginning of the game, so any number of consecutive heads are possible.

A breakdown of the first four possible stakes is listed in Table 1 below. Table 1 can be extended infinitely to the right to accommodate greater series of consecutive heads. The first row indicates the number of coin flips before the coin lands on tails and the stake is paid out. The second row indicates the series of coin flips achieved before the game pays out. The third row shows the probability of achieving the series of coin flips indicated in the second row. The fourth row indicates the payout of the game for the given set of coin flips indicated in the second row. The bottom row is the product of the third and fourth row. Summing the expected value of each individual coin flip (found on the bottom row of the table) across an infinite number of coin flips indicates the expected value of winning this game is infinite.

Table 1: St. Petersburg Payouts

<table>
<thead>
<tr>
<th>Rounds</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coin Flips</td>
<td>T</td>
<td>HT</td>
<td>HHT</td>
<td>HHHT</td>
</tr>
<tr>
<td>Probability</td>
<td>0.50</td>
<td>0.25</td>
<td>0.125</td>
<td>0.0625</td>
</tr>
<tr>
<td>Payout</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>Probability times Payout</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

The problem introduced by the paradox is that most people would not be willing to wager a sizeable sum of money to play this game, even though it has an infinite expected value. As a solution, Bernoulli (1954) proposed a concave utility function such that marginal utility decreases as the value representing the final state of wealth increases. The intuitive interpretation of the concave utility function is that greater wealth values bring smaller marginal increases in utility. For example, an increase of 100 from a starting wealth of 100 is valued with higher utility than a similar increase of 100 from 10,000.

A concave utility function restores some sensibility to the St. Petersburg game and brings about a prediction of risk aversion. Bernoulli’s modification to the payout line implies that even though the underlying value of the win increases in a doubling fashion indefinitely, the perceived utility of larger wins does not double correspondingly. Thus while the probability of each successive coin flip diminishes by half, the corresponding change in utility less than doubles. With a concave utility function, the sum of the bottom line converges to a finite value. As the sum of the expected values converges, risk aversion is observed such that players will settle for a single finite payout as a certainty equivalent to the prospect of playing the game.
Risk aversion serves as a tool in predicting what decisions people will make with regards to risky outcomes. Given two options, one certain and one based on a risky outcome, risk aversion predicts that if the actuarial value of both options is equivalent, people will choose the sure thing. Expected utility, including a concave utility function implying risk aversion, has been interpreted as a normative indicator of rational behavior. The notion of utility as defined as expected utility serves as a backdrop for a discussion of the value a customer experiences through buying an insurance contract.

Buying insurance is in line with the way a gambler assesses the utility associated with risky prospects. The insurance buyer is motivated when purchasing a contract by the value the insurance will provide in covering risky outcomes. A major focus is placed on the benefit of coverage the buyer will receive. The ability to gauge the value of insurance at the time of purchase is difficult to identify for the interested buyer. Finding the right level of coverage at the right price is important for the average consumer, mainly to avoid the risk of substantial financial loss. However, where insurance costs may be high or prohibitive, the savvy buyer will be wary of spending more on a given policy than its worth.

Learning more about a method of measuring the value of an insurance policy for the buyer, to score the utility of a given choice based on risky outcomes, will aid in providing confidence to the customer throughout the insurance buying process as well as giving the insurance industry a tool to provide for the need of its market. For example, a customer interested in purchasing insurance may be able to obtain one or more quotes for the premium required to maintain a given policy but may lack the experience to evaluate the probability of experiencing a claim and the likely range of severity if such a claim occurs. The lack of experience or claims data can cause uncertainty and lead to apprehension about purchasing insurance. A possible outcome is that the potential buyer does not purchase insurance that they would have otherwise found valuable.

Another outcome could be that the buyer does purchase insurance, but at a coverage level below what they would have chosen if they had more information related to the economic value that specific insurance policy offered. Yet a third possibility is that the buyer does purchase an insurance contract, but lacking confidence that the insurance policy offers what the premium is worth, the buyer remains “in the market” for that same type of insurance with a willingness to change insurers when a different premium quote is made available.

However, if the insurance buyer is provided with a level of confidence that the insurance policy purchased provides some understandable measure of economic utility, then they may be more willing to purchase the full amount of insurance coverage to accommodate their risk profile and less likely to shop around for new insurance after the coverage is initially purchased. The ability to settle the value of insurance policies on the basis of the utility the customer experiences is a useful problem to solve for customers, agents, and insurance companies.
2. Literature Review

How people evaluate decisions based on risky outcomes has been a topic of interest for centuries. How these decisions are made affect the way people buy insurance, engage in gambles, or invest in variable rate securities. One issue to be addressed is how insurance buyers’ attitudes shape the decision making when purchasing an insurance contract. An area of interest for the investigation of insurance customer sentiment is the degree to which the brand name of the company selling insurance affects the confidence the buyer has throughout the insurance buying process.

Taylor and Murrey observed that when less information is available to the customer, perceived risk involved with the purchase of insurance increased (1982). They concluded that respondents favored buying from a well-known company as a risk reliever (a device or action used to decrease the probability that a purchase will fail or shift the type of loss to one which the buyer has more tolerance). Their findings support the idea that an insurance company, by its very reputation, provides confidence in the insurance buying process. (Taylor & Murray, 1982). These findings support the notion that providing more information up front adds to the confidence the buyer experiences when purchasing insurance. This is consistent with the asymmetric information theory.

Bernoulli (1954) proposed what is known as expected utility theory which provides a basic understanding of how people make decisions under risk. In his discussion of decisions made under risky outcomes, Bernoulli identifies rules on what sorts of choices would be advised or ill-advised based on the values of the payouts and the underlying probabilities. His work focuses on the consideration of the utility of any given prospect as a determinant of the valid measurement of the value of a risk. Through a discussion on lotteries, Bernoulli identifies a feature of decision making where marginal increases in wealth offer diminished increases in utility for higher base states of wealth. The lower marginal utility predicted by Bernoulli’s expected utility theory is described as a concave utility function and provides the grounds for identifying risk aversion as a component of how people evaluate prospects. Bernoulli’s observations of expected utility and risk aversion have aided the understanding of insurance purchases.

The idea of risk aversion contributes to the theory that actuarial fair value is not the only factor considered when it comes to making choices governing risky outcomes. Mossin (1968) has developed formulas that use principles of risk aversion to show the viability of insurance as a valuable idea for the buyer. The advantage of insurance to a customer is found on the notion that a customer may pay a small premium up front to alleviate the risk of a larger loss at a later time.
The value of the insurance contract depends on whether the premium is commensurate with the actuarial fair value of the loss associated with the risk. The notion of risk aversion, which indicates that people will prefer a certain option over a risky one with the same actuarial value, establishes a basis of rationality, where predictability translates into reasonability, for the decision to purchase an insurance contract.

However, risk aversion does not adequately describe all consumer behavior when it comes to buying insurance. According to Schwarzc, “It has become increasingly clear in recent years that risk aversion is a remarkably poor explanation for how and why individuals purchase insurance” (2010, p. 557). Schwarzc describes several of what he terms “insurance demand anomalies,” such as individuals displaying little interest in purchasing catastrophe insurance, willingness to purchase insurance against small financial risk, and preference for low deductibles.

One explanation for the insurance demand anomalies is that the observed decisions represent an error on the part of the buyer. As explained by Schwarzc, “According to the mistake hypothesis, consumer insurance decisions reflect the fact that time is scarce, cognitive resources are finite and information limited” (2010, p. 562). Informational asymmetries between insurance companies and customers arise due to the level of expertise required to evaluate the risk and rating of insurance products. Insurers base their rates on experience tables, expense loading, and profit margins that are not publicly available to policyholders and applicants for new policies.

Schwarzc (2010) describes another way customers display decision making that deviates from choices made purely on actuarial fair value and expected utility as seen in how they make their decisions to select a deductible or to purchase an insurance contract at a set deductible. Classical theory predicts policyholders should favor higher deductibles as lower deductibles only serve to insure against small financial risks.

According to Schwarzc, higher deductibles disproportionately reduce the cost of insurance. Higher deductibles reduce moral hazard by requiring policyholders to pay the first dollars of a loss. They also reduce adverse selection since lower deductibles are disproportionately valuable to high-risk individuals. Due to disproportionality, the marginal cost of lower deductibles is up to four times higher than higher deductibles, yet a majority of consumers tend to select the lower deductibles.

In order to provide a standard to rate the rationality of customer behavior when setting an insurance deductible, it is worthwhile to identify what an optimal strategy would look like for selecting a deductible. Head (1965) provides a three-parameter model which offers an optimizing solution for the corporate buyer of insurance in the selection of an insurance deductible which may be used as a guide for identifying an optimal strategy for the individual.
According to his corporate buyer model, the basic rule is to choose that deductible which maximizes premium savings over loss assumption costs, provided that the deductible does not exceed the financial capacity of the firm. He notes that insurance buyers are not actuaries and that the relevant standard for predictability, which will guide the insurance buyer’s confidence in the credibility of their own experience, is guided by what the buyer’s nervous system is willing to take (Head, 1965). For an individual in the market for insurance, the insurance company will have its own company experience on which it bases its premium rates which may be more robust and credible than the buyer’s personal experience. As a result, the insurance buyer and the insurance seller may have differing attitudes as to what constitutes an optimal solution based on the predictability of experience available.

3. The Prospect Theory

Ultimately the decision to purchase insurance and select the features of an insurance contract reflect decision making under risk. The insurance contract represents the transfer of a known payment in the present to mitigate probable losses in the future. The prospect theory presents a contemporary set of descriptions of how individuals make choices like purchasing an insurance contract (see Kahneman and Tversky, 1979; Tversky and Kahneman, 1992).

Offered as a solution to critiques of expected utility theory, prospect theory provides an alternative model of decision making under risk. It focuses on the fact that people perceive outcomes as a change in states (gain or loss) as opposed to evaluating the final position of welfare. A resonant observation of Kahneman and Tversky’s prospect theory (1979) is that losses have a greater negative impact than gains of equal measure in the way people experience situations. The phenomenon is known as loss aversion.

Kahneman and Tversky’s paper included an experiment where subjects were asked to choose between a payout of 6000 with probability 0.25 or 4000 with probability 0.25 and 2000 with probability 0.25. In their study, 82% of respondents chose the 4000 and 2000 option. In the same study, on a different question where signs were flipped, 70% of respondents chose -6000 with probability 0.25 over the combined choice of -4000 at probability 0.25 and -2000 at probability 0.25. The results led Kahneman and Tversky to conclude “These preferences are in accord with the hypothesis that the value function is concave for gains and convex for losses” (1979, p. 277).

The concave/convex curve implies risk-averse behavior in the realm of gains but risk-seeking in the realm of losses. Also, the value function for losses is observed to be steeper than it is for gains, implying loss aversion. Based on an observation of the value function for gains and losses, they concluded that “losses loom larger than gains” in the minds of people faced with risky decisions (Kahneman & Tversky, 1979).
The idea of loss aversion is an influential feature of the formulas developed in the cumulative prospect theory to evaluate the utility provided by any given prospect (Tversky & Kahneman, 1992). Loss aversion makes unique predictions for loss based models, such as insurance, as opposed to prospects with only positive outcomes such as lotteries.

The prospect theory identifies the effect where people underweight outcomes that are probable compared to outcomes that are obtained with certainty. The different treatments of uncertain prospects versus certain prospects are seen to contribute to risk aversion in choices involving sure gains and risk seeking in choices involving certain losses. Prospect theory develops a theory of choice where the value is assigned to gains and losses as opposed to the final welfare state. Features of the model include a value function that is steeper for losses than gains and decision weights that replace the underlying probabilities when evaluating a risky choice.

Their paper on prospect theory includes other examples of ways in which subjects responding to hypothetical questions provide answers that go against predictions made by expected utility. The authors refer to one example as the certainty effect, a phenomenon where people overweight outcomes that are considered certain over outcomes that are probabilistic (Kahneman & Tversky, 1979).

The difference in the weighting of probabilities by certainty is seen to contribute to risk aversion in choices involving sure gains and risk seeking in choices involving certain losses. Understanding the certainty effect is helpful in making comparisons between cash flows that are considered a sure thing, compared with those which are merely probabilistic. The model presented in this paper is supported by their work on the certainty effect. The model identifies a single payment, taken as a sure thing, and compares to risky prospects with probabilistic outcomes.

The prospect theory also replaces pure probabilities with decision weights which vary as the underlying probabilities approach zero or one. Kahneman and Tversky (1979) indicate decision weights are lower than their corresponding probabilities, except in very low probabilities.

Using Tversky and Kahneman’s (1992) experimentally derived values, Bromiley (2010) has illustrated situations where prospect theory predicts risk averse and risk-seeking behavior. For a series of payouts from -20 to 20, he calculates a subjective value, $v(x)$, found in equation (1) for a given payout and the value of a gamble calculated as the mean of the subjective values of the outcome increased and decreased by 5. For example, the value of a certain payout of 10 is compared to a risky option with an equal probability of obtaining 5 or 15. Where the value of the certain payout exceeds the value of the gamble, risk aversion is implied. Based on the parameters shown above, the certain choice is preferred to the risky choice for all payouts of -2 to 20. Only outcomes where the certain option incurs a loss lead to risk-seeking behavior.
The highest degree of risk aversion is indicated when the certain option is 3 and the highest degree of risk-seeking occurs when the certain option is -5. As the payouts increase above 3 and decrease below -5 the risk premium implies greater risk neutrality.

The implication is that decision makers show risk aversion in the positive realm, and risk seeking in the negative realm, with a complex interaction for decisions involving mixed results (some positive and some negative outcomes).

Bromiley’s approach provides the groundwork for comparing a single payout with multiple payouts where the single payout is considered certain and the other payouts have probabilities that are determined a priori. Bromiley’s approach calculates a score without units for both certain and uncertain prospects. Prospects with higher scores are considered preferred over prospects with lower scores. This paper follows a similar methodology from the cumulative prospect theory.

Tversky and Kahneman’s cumulative prospect theory (1992) indicates risk aversion for gains and risk seeking for losses of high probability with risk-seeking for gains and risk-aversion for losses of low probability. An advantage of aligning a method for summarizing the utility of insurance with the cumulative prospect theory is that the groundwork has been laid for analysis of losses with low probability.

3.1 Single Payout Utility Equivalent (SPUE)

In order to judge the preference of one prospect over another, the cumulative prospect theory incorporates two functions which, when evaluated together, provide a numeric value that can be used to order prospects (see Tversky & Kahneman, 1992).

To evaluate a prospect, decision weights, symbolized by \( w(p) \) as a function of the underlying probability \( p \), is multiplied by the subjective value, \( v(x) \), of an outcome \( x \) to determine an overall value, \( V(x) \), of an edited prospect (one that has already been evaluated under certain constraints by the evaluator) to create a final ranking of choices available. Prospects with higher numeric values of \( V(x) \) are preferred to lower valued options. They calculate a subjective value of payout, \( x \), as follows:

\[
v(x) = x^\alpha \text{ if } x > 0, \quad \text{and} \quad v(x) = -\lambda(-x)^\beta \text{ if } x < 0 \quad (1)
\]

The decision weights associated with each probabilistic outcome are identified by the function below.

\[
w(p) = \frac{p^\kappa}{(p^\kappa + (1-p)^\kappa)} \text{ if } x > 0, \quad w(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)} \text{ if } x < 0, \quad \text{and } w(0) = 0 \quad (2)
\]

Tversky and Kahneman conducted an experiment in order to estimate the parameters in the value and weighting functions (1) and (2). Subjects were asked to rank their preferences for risky prospects. From their experiment, median values for the parameters were found. The median exponent for the value function was 0.88 for both \( \alpha \) and \( \beta \). The median \( \lambda \) was 2.25. The median \( \kappa \) and \( \delta \) were 0.61 and 0.69 respectively (Tversky & Kahneman, 1992).
Though not focused on the specific quantitative effects, the authors used a nonlinear regression procedure to obtain values for the parameters of eq. (1) and (2). If a method is provided that can find appropriate values for the parameters $\alpha$, $\beta$, $\lambda$, $\kappa$, and $\delta$, the equations of cumulative prospect theory provide a sound basis for identifying an objective measure of utility experienced by a subject when faced with the opportunity to either accept or mitigate the gains and losses associated with a given prospect.

While the measure yielded by applying the equations of cumulative prospect theory does not translate directly into any currently observed units of measure, it does provide an indication to compare two sets of choices based on differing risks and to identify which choice should be preferred by the subject. Additionally, for any single probability-payout combination there exists a unique payout $S$ such that the utility measure of $S$ for certainty (evaluated with $p = 1$) is equal to the valuation of the probabilistic choice. The value $S$ represents a single payout utility equivalent (SPUE) which the subject would be indifferent to exchanging for the choice involving the probabilistic outcomes. The equation for finding a single payout $S$ given a corresponding probabilistic payout $x$ at probability $p$ is shown in eq. (3).

$$v(S) = v(x) \ast w(p)$$  \hspace{1cm} (3)

For example, if $x$ is positive, the value of $S$ may be calculated via eq. (4), as

$$S = e^{\frac{(v(x) - w(p))}{\alpha}}$$  \hspace{1cm} (4)

By applying the principles of cumulative prospect theory, the concept of the SPUE can be expanded to include prospects with multiple possible payouts with different probabilities. Here an assumption is made that when two payouts are offered at different probabilities, the overall value of the prospect is calculated by combining like payouts across probabilities.

For example, if a prospect would pay out 200 at a probability of 0.30 and 450 at a probability of 0.5, the payout of 200 would be considered to occur at the combination of probabilities 0.3 and 0.5 and an additional 250 (calculated as 450 minus 200) would be observed to occur at probability 0.5. The value $V$ would be evaluated by the following equation.

$$V = v(200) \ast w(0.30) + v(250) \ast w(0.5)$$

So for a given set of payouts $x_i$ with corresponding probabilities $p_i$, the SPUE can be calculated as $S$ in eq. (5).

$$v(S) = \sum_i v(x_i) \ast w(p_i)$$  \hspace{1cm} (5)

The SPUE for a non-discounted cash flow is found by first applying the formulas for Tversky’s and Kahneman’s cumulative prospect theory across all outcomes and probabilities to arrive at a utility value $V(x)$ of the expected cash flows. Then a single payment utility equivalent $S$ is solved by using the same value function and substituting a probability 1 for the weighting function. The dollar amount obtained by solving for $S$ represents the cash equivalent of the utility being purchased under the insurance contract.
Further, a SPUE can be calculated for a series of cash flows that take into account the time value of money for cash flows occurring in the future. For a discounted cash flow, first the value for each time period is calculated using Tversky and Kahneman’s value function, then the value is discounted back to the time of purchase by the time value of money. Then for the valuation of the SPUE, the discounted values are replaced for v(x) in eq. (5).

For example, the SPUE of a 3 year warranty on a product sold for $1000, and a probability of failure of 0.005 in each year, covered for one full replacement in the three year warranty period (calculated as paid at the end of the year) with a 5% annual discount rate may be evaluated as follows. The probability of failure in a given year, and the discounting at the end of each year are shown in Table 2 below. The probability that the product will fail in a given year for the first three years is 0.005, 0.004975, and 0.00495. The discounted amount of the loss at the end of each of the first three years is -935.41, -890.82, and -848.39.

Table 2: Three Year Warranty

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure Probability</td>
<td>0.005</td>
<td>0.004975</td>
<td>0.00495</td>
</tr>
<tr>
<td>Discounted Loss</td>
<td>-935.41</td>
<td>-890.82</td>
<td>-848.39</td>
</tr>
</tbody>
</table>

The value of the utility of the warranty is summarized as the sum of the products of the discounted cash flows by the corresponding decision weights. The value of -848.39 is realized in all years where the product fails, thus it is calculated as the sum of the probabilities of each year which is 0.014925. The additional value of -42.43 (calculated as the difference -890.82 minus -848.39) is incurred when the product fails in the first or second year; the additional loss of -42.43 is calculated at the combined probability of failure in years one and two which is 0.009975. Finally, the additional loss of -44.59 (calculated as the difference of -935.41 minus -890.82) is incurred only if the product fails in the first year. A calculation of the SPUE for the illustrated warranty is shown in the following equation.

\[ v(S) = v(-848.39) \times w(0.014925) + v(-42.43) \times w(0.009975) + v(-44.59) \times w(0.005) \]

Here, the single payment utility equivalent is -$47.84. What this number implies is that on average, were the terms fully understood, a buyer would be indifferent to accepting a certain loss of around $48 or accepting the risk of loss at the given probabilities and discounting.

Note that the utility values calculated in the cumulative prospect theory serve as comparative values only, and are not formatted to any unit or denomination. The conversion of the measure of utility provided by the cumulative prospect theory’s formulas to an SPUE is necessary to convey a dollar value of benefits incurred.

Also of note is that the parameters presented in Tversky and Kahneman’s paper on cumulative prospect theory represent data derived from a survey conducted on a select group and may or may not fit well for any single individual. Where the means are available, the communication of an SPUE may be more individualized. It is anticipated that for any given
insurance marketing where an SPUE is incorporated, the parameters of the prospect theory formulas will be recalibrated via a survey with hypothetical scenarios to best capture the idiosyncratic experience of the potential buyers of that type of insurance. By recalculating the parameters for each individual the utility presented by an SPUE can be tailored to the risk attitudes of the specific person looking to purchase a specific type of insurance.

3.2 Who Should Calculate a Single Payout Utility Equivalent?

A question arises, as to who is best suited to perform the evaluation of the parameters involved in the cumulative prospect theory functions. A third party to the relationship between the insurance provider and the insurance buyer seems best suited to provide the means for calculating the measure of utility provided by an insurance contract without creating a conflict of interest. It is not difficult to see that an insurer will be motivated to persuade a potential customer that purchasing insurance is for their own economic interest.

It is foreseeable however that an insurance agent could administer a survey and submit the results to a third party software solution in order to provide the SPUE to the customer within the context of a real-time conversation about insurance options. The SPUE is proposed as more of a comparative tool, not so much for the purpose of securing a sale than for giving a potential customer a means to line up one insurance product or option against another. Using it merely to promote insurance sales misses the point. An insurer is a party on the side of the informational asymmetry who has data on the underlying probabilities of the risks being insured for. Since the insurer owns the tables identifying the risk on which the insurance premium is calculated, the insurer is the party in the relationship able to make meaningful evaluations of the likelihoods used in the prospect theory decision weighting functions.

Does the insurer have the capacity to present a measure of utility in such a way that the practice can benefit the customer and at the same time provide that potential buyer with a sense of confidence that conflicts of interest on the part of the insurer offering advice are minimized? If the insurer supplies the probability tables to a third party solution, such as a software arrangement that takes the calculation out of the insurer's hands, a degree of impartiality may be maintained. The following presents one approach that may be taken to present to a customer some measure of the subjectively experienced value under consideration for purchase.

For a hypothetical scenario, imagine an insurance agent with multiple lines of product offerings home, auto, and life. After conducting a survey of three to five questions where the insurance customers indicate their preferences for selected prospects at known probabilities, through the use of calculation tools provided by the third party regarding relative risk scores for the customer, the agent will be able to present the customer with single premium utility equivalents for each line as well as different levels of coverage. Our agent may come to the buyer with a statement,
“Given your responses to certain risk-based questions, and amounts that you have stated you would be indifferent between accepting a single payment or risky cash flows at a known probability of outcome, a risk profile has been generated to estimate the utility you receive from varying types of insurance. Coupled with company experience tables used in the pricing of insurance premiums, your risk profile indicates you will experience a level of utility commensurate with a single payout in dollars for the following selected insurance plans. Note the following Single Payment Utility Equivalents are not the premium being charged for the insurance policies, but rather a measure of these products’ economic utility converted to equivalent dollars that they will bring to you. For a $150,000 accidental liability on your vehicle with the standard policy coverage details the SPUE is $645. A homeowner’s policy with $400,000 accidental loss coverage has an SPUE of $1196. At your request provided are ratings for two 10 year term life insurance policies. The first for $500,000 coverage has an SPUE of $680. The second for $750,000 is $1019.”

The agent could then engage in a discussion of premium rates and engage in a talk about the types of risk being covered for and the utility the agent can provide through the products offered.

Though it may be difficult to convince the customer that the agent presenting the SPUE is impartial to the decisions implied by the utility score, it may be possible to establish some standard of objectivity in the calculation and delivery of such a measure. If the concept is deemed to provide meaningful value to the insured, and possibly aid in the sale of insurance policies, players in the industry may desire to have a third party moderate the SPUE process. While the insurance industry is usually resistant toward government regulations, it is conceivable that a third party to the insurance company and customer relationship could be encouraged to oversee the administration of the SPUE process if it would add credibility and transparency to insurance buying.

Having established a method for communicating the utility of a given insurance product with high-value payouts and low probability, a similar approach may be considered for covering lower value losses such as is encompassed by product extended warranties. The first item of note is that the sale of most product extended warranties take only a few seconds from the moment the warranty is offered to the moment of purchase, coinciding with the point of sale of the product to which the extended warranty is set to cover. Since the approach of surveying each customer to determine a personalized risk profile would likely lengthen the time required to sell any individual extended warranty without providing any additional value to the customer or seller, a more generalized approach to informing customers of the utility gained by purchasing a given extended warranty can be taken.
If the risk parameters for the cumulative prospect theory functions are assumed, the SPUE can be calculated for any given product provided the manufacturer has some expectation of that product’s failure rates. Further, to provide a standardized measure of the value of all extended warranties across different products and values, a warranty utility factor, calculated by dividing the SPUE by the actual cost of the warranty, can be determined for each warranty.

Between two warranties, the product with the higher warranty utility factor can be said to provide greater financial utility per dollar cost of the extended warranty. The advantage of a standardized factor is that consumers may become familiar with the concept and acquire a sense of comparative value across all extended product warranties. A sense of understanding about the utility value of the extended warranty to be purchased can offer more confidence to the customer in the decision-making process of whether or not to opt for additional coverage on a given product being purchased. Without a standardized measure, the consumers find themselves uncertain as to how much the pricing of an extended warranty overstates or pads the risk of defect covered by the contract.

To ensure similar treatment for products from different manufacturers and sold through different outlets, the role of calculating and certifying the warranty utility factors is, similar to the insurance SPUE, best suited for a third party rating agency. Manufacturers may submit the failure rates of their products to the third party rating agency which would then be able to calculate a utility score for an extended warranty at a given duration and price. By subjecting all sellers of extended warranties to the same process, uniformity may be understood for all extended warranties across products and markets. At the point of sale, a seller need only state the duration, warranty utility factor, and price.

3.3 Application of the SPUE

One test of the prospect theory model is to examine the empirically solved parameters in common risk decision scenarios to see if the model conforms to usual experience or if the parameters and model predict unexpected behavior. A way to conduct a sensibility test will be to plug in standard insurance values and risk probabilities to see if the utility of a certainty equivalent as predicted by the prospect theory lines up with the range of costs currently charged to insure against a specific risk.

A life insurance risk is considered here as an example. Considering a policy for a 50 year old is listed at a 0.00068 mortality risk for the first year by the Society of Actuaries 2017 Commissioner’s Standard Ordinary Table, the expected converted utility from the prospect theory model following the established parameters to cover the risk of losing 500,000 yields the same converted utility as a certain payment of $1,494. In other words, the perceived utility at risk by the hypothetical individual captured by Tversky and Kahneman’s parameters is roughly equivalent to $1,494 per year. The prediction is supported by the finding that the current online market quotes available for a 50-year-old 10 year 500,000 term policy are priced
at $58-65 a month and further justifies the expectation that an insurer may expect to sell policies, with loading, at a price above the actuarially fair premium (no interest discounting) of $305 for one year of coverage.

For a second example, an auto insurance risk is considered. Using a rule of thumb that the average adult experiences an auto insurance claim once every 17 years, and estimating an average claim severity of $10,000, the prospect theory model predicts a converted utility of the risky portion equivalent to the converted utility of a certain loss of $925 for one year. Compare $925 to an auto premium quote of $246-$459 per six months and the SPUE does not fail the sensibility test.\(^1\)

Higher values in the Kahneman and Tversky model make sense, considering that the market price for financial products is expected to follow standard supply and demand dynamics. Competition among suppliers has lowered the market price for insurance below the dollar value that can be substituted for equivalent utility. The prediction is that where the insurance market has a large number of suppliers, the market value of insurance products will approach marginal cost or actuarial fair value.

Left with only market prices for insurance, the value of the utility of insurance products has been obscured by the insurance quoting process. The consumer, faced with substantial informational asymmetries, does not have an easy way to evaluate how much gain they are receiving from the insurance contract. This provides a creative opportunity for the insurance seller to inform the buyer of how much value the product provides.

The contribution this paper intends to make is to suggest that insurers, or some third party intermediary, offer some information calculated from prospect theory to inform insurance buyers of what experiential value they are earning by buying a given insurance product. For a given insurance product, the customer is presented with the SPUE to express a cash value of the experience a consumer would gain by purchasing this product. As indicated in the examples provided earlier, where the SPUE is greater than the market price, the insured is encouraged to buy. Also, when considering related products, this approach provides the buyer with some consistent method of comparison to assess the underlying benefits of each product as anticipated by the prospect theory model. A key element of the SPUE is that it takes into account how the insurance buyer would evaluate the insurance policy if they had available the same data as the insurer.

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\(^1\) The premium quote range was obtained by querying multiple online insurers for a 2010 Kia Sedona with a 41 year old female driver.
3.4 SPUE and Deductibles

An aspect of establishing an insurance contract is to determine a deductible in the case of an insurance claim. Deductibles determine the amount of a claim a policyholder must cover before an insurer pays out on a claim. Higher deductibles reduce insurance premiums as the insurer has less exposure for any given incident that would qualify as a claim. Additionally, deductibles serve to reduce moral hazard in the insurance environment as policyholders are still liable to experience some level of loss when a claim occurs. Higher deductibles reduce moral hazard and hence premium compared to lower deductibles. An insurer will take interest in how customers make decisions to set the deductible when establishing an insurance contract. Demand for certain deductibles affects pricing and loss profiles.

Currently, deductible decisions are offered in different ways depending on the type of insurance coverage contracted. Auto and homeowner’s insurance frequently offer flexible options where the policy applicant selects a deductible level from a menu and premiums are calculated based on the insurer’s exposure. Health insurance deductibles are typically set on a per plan basis where the person to be insured either opts into a plan at the set deductible and pricing or finds an alternative plan. Health insurance customers do not typically have an option to tailor the deductible to their own individual needs. Life insurance does not by nature include deductibles as the contract is set at a specified death benefit which is paid out in the event of a claim without establishing loss severity.

As stated earlier, classical insurance theory predicts diminishing marginal utility for lowering deductibles which only cover small amounts of loss at low probabilities. According to classical insurance theory, insurance makes sense when it covers large losses for a low fixed premium. Yet field data shows that customers often opt for lower deductibles. One hypothesis for the anomaly in the behavior to opt for lower deductibles is that insurance buyers overestimate the likelihood of a claim when considering only the additional coverage provided by a lower deductible.

Additionally, the practice of settling on a deductible amount before quoting the premium on insurance may obscure from the customer how much the premium increases for a corresponding lowering of the deductible amount. One option which may provide more awareness for the insurance customer would be to provide the customer with an SPUE for an insurance policy at each level of deductible. The SPUE approach has the advantage of offering comparability across multiple lines of insurance and providing a single number measure of utility for the customer. In providing this information to the customer, the agent may be able to illustrate that lower deductibles do not offer a proportional increase in utility corresponding to an increase in premium. This would address certain insurance buying anomalies discussed by Schwarcz (2010) in Section II above.
Use of the SPUE may change insurance buying behavior to incorporate higher deductibles per policy in general. It may be argued that higher deductibles are better for the insurance market all around as they reduce moral hazard on the part of the insured but still allow for coverage of large or catastrophic loss in a risky environment. Insurance providers may offer insurance coverage for large losses while avoiding the costs associated with minor claims.

4. Conclusion

This paper considers ways in which people choose to purchase insurance policies to mitigate risk, and proposes a new way of presenting the value of a policy that provides greater communication transparency. While insurers and agents are motivated to justify the prices of their policies as a good value for the customer, there are problems with how a customer may compare or evaluate the economic good received from a given policy.

We discussed the expected utility theory and the prospect theory which illustrate how models of decision making under risk have developed. Relying on Tversky and Kahneman’s (1992) functions employed in the cumulative prospect theory, this paper proposes a method of calculating a dollar equivalent amount that represents the utility of any given insurance policy, the single payment utility equivalent (SPUE).

SPUE presents a new option for the agent selling an insurance policy to communicate to the customer. Given widespread use, SPUE may provide a common sense approach to how willing the buyer would be to purchase an insurance policy if they are aware of the underlying probabilities that were used in the pricing of the policy’s premium.

This paper also presents several examples of applications of the SPUE different types of insurance, at varying values of coverage including life, auto, and homeowner’s insurance as well as product warranties. Additionally, the SPUE concept can help insurance customers decide upon a deductible as a feature of their policy. In the end, the measure of the economic good described in this paper, the SPUE, when offered to potential customers, may make the insurance buying process more transparent.

References


