Optimal Setup Cost Reduction and Lot Sizing with Imperfect Production Processes and Quantity Discounts

Tien-Yu Lin,
Department of Marketing and Supply Chain Management,
Overseas Chinese University, Taiwan.
E-mail: admtyl@ocu.edu.tw

Abstract
This paper concentrates on an economic production quantity model with quantity discounts and imperfect quality, where the screening process occurs at the production duration. Specifically, this paper considers that the production rate is finite, which is unlike Porteus’ (1985) model, and the supplier offers the quantity discounts to stimulate the buyer ordering more quantities. Each item is inspected immediately after it is produced. Then, the imperfect quality items are stored at a specific warehouse with a lower holding cost rate compared to good items stored at another warehouse. With respect to the imperfect quality items, the manufacturer sells them for salvage value at a different market by the end of production process. The objective of this study is to determine the inventory lot size and the setup cost which is a function of capital investment maximizing the expected total profit. An efficient algorithm is then developed to find the overall optimal solution for the model. Numerical example is presented to illustrate the proposed model and algorithm. Managerial insights are also explored.

Key Words: Inventory, Imperfect quality, Lot sizing, Capital investment, Quantity discounts
JEL Classification: C 61, M11
1. Introduction

Manufacturing the right levels of product to satisfy customer needs have become an important issue for the enterprises. The production processes are assumed to be perfect and stationary in the traditional economic production quality (EPQ) models. However, in practice, this assumption may not be true because the product quality usually depends on the state of the production process. In general, the production process initially is under control and items produced may be perfect quality. As time goes on, the process may deteriorate as a result of which the items produced contain defectives or items that are of substandard quality (Rosenblatt and Lee, 1986). Several excellent researchers have paid their attention to the inventory models where the quality produced may not be good due to an unreliable production process. One of the pioneer papers in this issue has been studied by Rosenblatt and Lee (1986) in which they explored the effects of an imperfect production process on the optimal cycle time. In the same time, Porteus (1986) incorporated the effect of defect items into economic order quantity (EOQ) model studying the relationship between quality and lot size. Zhang and Gerchak (1990) explored a joint lot sizing and inspection policy for an EOQ model considering the random yield. Cheng (1989) proposed an EPQ model with a flexible random yield process and employed a geometric programming method to solve the developed model. Cheng (1991) formulated an EOQ model with demand-depend unit production cost and imperfect production processes. He further transferred this optimization problem as a geometric programming for solving. Bose et al. (1995) proposed an EOQ model for deteriorating items with a linear time-dependent demand rate in which the shortages and backlogging are allowed. Yano and Lee (1995) have made a comprehensive survey on these models. An interesting variant recently has been developed by Salameh and Jaber (2000) who extended the traditional EPQ model by considering that identifying imperfect quality items employs an inspection process. Ever since Salameh and Jaber’s (2000) model was developed, which considers the quality control and thus is more reasonable than the traditional EOQ/EPQ models, many excellent researchers have extended their models into the issue of EOQ/EPQ models with imperfect quality (for example, Chan et al., 2003; Huang, 2004; Eroglu and Ozdemir, 2007; Maddah and Jaber, 2008; Lin, 2010; Lin, 2013). In some recent works, Wee and Widyadana (2013) explored a production system for deteriorating items with stochastic preventive maintenance time and rework. Taleizadeh et al. (2014) developed an EPQ inventory model for multi products in a single machine with rework and interruption in process, which is different traditionally non-interrupt production process. Wee et al. (2014) investigated an EPQ model for imperfect quality items with non-synchronized screening and rework in which they use time intervals rather than ordering size and backorder size as decision variables. An EOQ with imperfect items which considers investing in the
speed of the quality control check is presented in Hauck and Voros (2015) and Lin et al. (2016). The authors in Moussawi-Haidar et al. (2016) integrated the inspection time into the economic production model with rework, and demonstrated the significant effect that the inspection time has on the results. They further considered a manufacturing process with random supply and a screening process conducted during and at the end of production. Al-Salamah (2016) developed an economic production quantity (EPQ) model for the case where the production process and inspection are both not perfect in which two settings for the sampling stage have been considered: destructive and non-destructive testing of the sample items. A comprehensive survey of the modified EOQ model extensions for imperfect quality items was explored by Khan et al. (2011b). Note that the above-mentioned models based on Salameh and Jaber’s (2000) work mostly dealt with EOQ models and seldom studied EPQ modes. However, in practice, the production capacity is finite and thus EPQ models play an important role for the inventory issue. We also note that imperfect quality EOQ models based on Salameh and Jaber’s (2000) model assumed the retailer performs the screening processes for items after an order received. This assumption may not match EPQ cases because each item is immediately screened after it is produced. The defective items are then usually stored in a different warehouse from the good items to separate the tracking of costs and quantities for the defective items (Paknejad et al., 2005; Wahab and Jaber, 2010; Lin et al., 2016). Many semiconductor industries provide good examples. With this consideration, the holding cost for a unit of good items per period is larger than for a unit of defective items per period. Thus, an EPQ model, where the screening process occurs at the production duration, with different holding cost is worth study.

Recently, the issue of setup cost reduction in production systems received a considerable attention (e.g., Porteus, 1985; Hong and Hayya, 1993; Sarkar and Coates, 1997; Moon et al., 2002; Hou, 2007) in which the researchers have recognized setup cost can be controlled and reduced through various efforts such as specialized equipment acquisition, procedural changes and learning effects. Porteus (1985) is the first pioneer to develop a framework encompassing not only altering the setup cost but the demand rate for investing in reducing setup cost in the EOQ model. Porteus (1986) further introduced the investing options into the EOQ model to explore the effects of process quality improvement and setup cost reduction on the lot size. Ever since Porteus (1985, 1986) showed his ideas, several extensions were built. For example, Billington (1987) developed an EPQ model with the setup cost parameter replaced by a function of capital investment. The economic benefits of reducing setup cost and improving process quality by simultaneously investing in new technology was studied by Keller and Noori (1988), Hong et al. (1992), and Hong and Hayya (1995). Several relationships between the setup cost level and the expenditure of capital investment have been discussed by Paknejad et al. (1990), Kim et al. (1992), and Hou (2007). Unlike previously
studies, some researchers (Andijani, 1998; Wu, 2002; Goyal and Gunasekarn, 1990) further explored setup cost reduction in multi-stage production inventory systems. Other related studies on setup cost reduction include Huang et al. (2010), Voigt and Inderfurth (2011), Sarkar and Majumder (2013); Sarkar and Moon (2014) and references therein. We note that the above literatures indicate the setup cost can be reduced at an added investment and thus has significantly influence for decision making. Thus, considering the relationships between the amount of capital investment and setup cost level is very important when capital investment strategies are employed.

Based on the above arguments this paper studies an EPQ model with quantity discounts and imperfect quality, where the screening process occurs at the production duration. Specifically, this paper considers that the production rate is finite, which is unlike Porteus’ (1985) model, and the supplier offers the quantity discounts to stimulate the buyer ordering more quantities. Each item is inspected immediately after it is produced. Then, the imperfect quality items are stored at a specific warehouse with holding cost rate $f_d$ (a unit of defect item per period, expressed as a fraction of dollar value) compared to good items stored at another warehouse with holding cost $f_s$ (a unit of good item per period, expressed as a fraction of dollar value), in which $f_s > f_d$. With respect to the imperfect quality items, the manufacturer sells them for salvage value at a different market by the end of production process. The objective of this study is to determine the inventory lot size and the setup cost which is a function of capital investment maximizing the expected total profit.

2. Notations and Assumptions

The following notations are used throughout this paper to develop the mathematical model.

2.1 Notations

$D$ demand rate

$K$ setup cost for each production run

$M$ production rate, $M > D$,

$Q$ lot sizes for each production run, a decision variable

$K_0$ original setup cost for each production run prior to investment

$K$ nominal setup cost, a decision variable

$c_j$ unit purchase (production) cost of $j$th level corresponding to the cost discount structure

$f_s$ holding cost rate for a unit of good item per period, expressed as a fraction of dollar value and $f_s > f_d$.

$f_d$ holding cost rate for a unit of defect item per period, expressed as a fraction of dollar value and $f_d < f_s$.

$p$ percentage rate of defective items in $Q$
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s  unit selling price of perfect items
v  unit salvage value of scrap items, v<s
x  unit screening cost
t₁  production run, t₁=Q/M
φₖ  capital investment required to reduce setup cost from K₀ to K
α  fraction of reduction in K per dollar increase in φₖ, α<i
T  cycle length

2.2 Assumptions

The assumptions in this paper are described as follows:

(1) The demand rate for items is known and constant.
(2) The holding cost for good items in a specific warehouse is higher than that of defective items in another warehouse.
(3) Each item is screened at the time of items as it is output from the production run, and all defective items are detected and then sold with a salvage value when the production cycle is over.
(4) Shortages are not allowed.
(5) The relationship between setup cost reduction and capital investment for each cycle time can be described by the logarithmic investment cost function. That is, setup cost, K, and the capital investment in setup cost reduction, φₖ, for each cycle time is described as follows:

φₖ = \frac{1}{\alpha} \ln \left( \frac{K₀}{K} \right)

(6) The supplier provides all-unit quantity discounts to the buyer. The cost scheme is listed in Table1 and cⱼ is the unit procurement cost for the jth level (i.e., if Qⱼ₋₁ ≤ Q < Qⱼ, then the unit procurement cost is cⱼ).

<table>
<thead>
<tr>
<th>j</th>
<th>Qⱼ₋₁ ≤ Q &lt; Qⱼ</th>
<th>cⱼ</th>
</tr>
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<td>0 &lt; Q &lt; Q₁</td>
<td>c₁</td>
</tr>
<tr>
<td>2</td>
<td>Q₁ ≤ Q &lt; Q₂</td>
<td>c₂</td>
</tr>
<tr>
<td>3</td>
<td>Q₂ ≤ Q &lt; Q₃</td>
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<tr>
<td>r</td>
<td>Qᵣ₋₁ ≤ Q &lt; ∞</td>
<td>cᵣ</td>
</tr>
</tbody>
</table>

3. Mathematical Model

In this section we develop a mathematical model with the total profit maximization objective for the investment cost of changing and the inventory related costs associated with
quantity discounts, by optimizing over both \( K \) and \( Q \). This model considers that items have imperfect quality and thus each item is inspected immediately after it is produced. Then, the imperfect quality items are stored at a specific warehouse with holding cost rate \( f_d \) compared to good items stored at another warehouse with holding cost \( f_s \), in which \( f_s > f_d \). With respect to the imperfect quality items, the manufacture sells them for salvage value at a different market by the end of production process. Quantity discounts are offered by the supplier to stimulate the buyer to order a larger amount. Because this paper deals with undiscounted costs, an opportunity cost per unit time is therefore charged. Thus, the production system with imperfect processes and different holding costs is illustrated in Fig. 1 in which \( T \) is the cycle time for each production lot and \( t_1 \) is a production run \((t_1=Q/M)\). Unlike Salameh and Jaber’s (2000) work, this paper assumes each item is screened as it is output from the production run (i.e. \([0, t_1])\). The defective items are sold at a different market for salvage value when the production run is over. Therefore, the cycle time \( T \) could be expressed as

\[
T = \frac{Q(1 - p)}{D}
\]

Figure 1. The behavior of the inventory level per cycle

From Figure 1, we know the maximum inventory level at the end of \( t_1 \) is \( Q(1-D/M) \). As the production run is finished, the defective items, \( pQ \), are then sold at different market and therefore the inventory level immediately drops to \( Q(1-D/M-p) \). Other explanations for Figure 1 will be introduced when they are needed.

Let \( TR_j(Q) \) and \( TC_j(Q,K) \) be the total revenue and the total cost per cycle corresponding to the quantity discounts schedule, respectively. \( TR_j(Q) \) includes the revenues from good and defective items and is given by
$TR_j(Q)=sQ(1-p)+vQp$

$TC(Q,K)$ consists of set up cost per cycle, screening cost per cycle, procurement cost, amortized capital cost, and holding cost. These costs are described as follows.

We know the cycle time $T$ for each production lot is composed of two parts ($t_1$: production run; $t_2$: non-production run) and the defective items are sold at the terminal time of $t_1$. Thus, the inventory quantities in the $t_1$ duration of are composed of good and imperfect quality items; the inventory quantities in the duration of $t_2$ are all good quality items. Therefore, the inventory holding cost could be obtained shown as below:

1. The holding cost occurs in the duration of $t_1$

In the production run, $t_1$, the items produced can be classified as perfect quality items and imperfect quality items after the screening processes and then stored them at different warehouse. The imperfect quality items with $p$ proportion of each production lot are sold when the production is over. Let $HC_d(Q)$ be the holding cost for imperfect quality items in the duration of $t_1$. Thus, $HC_d(Q)$ could be computed based on the triangular area of $(\text{beg})$ with proportion $p$ in Figure 1 and can be easily obtain by employing a geometric argument through the following:

$$HC_d(Q) = pc_j c_d \left[ \frac{Q^2 \pi}{2M} \right], \text{where } \pi = 1 - D/M \text{ and } \forall j = 1,2,\ldots,r \quad (2)$$

Alternatively, let $HC_g(Q)$ be the holding cost for perfect quality items in the duration of $t_1$. We have

$$HC_g(Q) = (1-p)c_j c_g \left[ \frac{Q^2 \pi}{2M} \right], \forall j = 1,2,\ldots,r \quad (3)$$

2. The holding cost occurs in the duration of $t_2$

We note that the screening processes for all items are finished at the end of $t_1$ and then the imperfect quality items are sold at that time. This implies all the items in the non-production run, $t_2$, are in the good quality condition. Let $HC_g(Q)$ be the holding cost for perfect quality items in the duration of $t_2$. Thus, $HC_g(Q)$ could be computed based on the triangular area of $(fgy)$ in Figure 1 and can be easily obtain by employing a geometric argument through the following:

$$HC_g(Q) = c_j c_g \left[ \frac{(\pi - p)Q^2}{2D} \right], \forall j = 1,2,\ldots,r \quad (4)$$

Other costs in each cycle could be described as follows:

(a) Setup costs:

The setup cost in a production cycle is

$Setup cost = K$ \hfill (5)
(b) Procurement (production) cost:

We assume the supplier offers the all-units quantity discounts to encourage the manufacture into ordering more quantities. This implies the procurement (production) price is some function of the quantity purchased. Thus, the procurement cost corresponding to the unit invoice cost $c_j$ for each production cycle is

\[
\text{Procurement cost} = c_j Q
\]

(6)

(c) Amortized capital cost:

The relationship between the reduction in setup cost and capital investment can be described as logarithmic investment function which has been adopted by Poteus (1985, 1986), Sarker and Coates (1997), Sarker and Moon (2014), Hou et al. (2016). That is, the setup cost, $K$, and the capital investment in setup cost reduction, $\phi_k$, can be shown as

\[
\phi_k = \frac{1}{\alpha} \ln \left( \frac{K_0}{K} \right), \text{ for } 0 < K \leq K_0
\]

Therefore, the amortized total capital cost for each production cycle can be presented as

\[
\text{Amortized capital cost} = \frac{1}{\alpha} \left[ \ln \left( \frac{K_0}{K} \right) \right] Q (1-p) \frac{D}{D}
\]

(7)

(d) Screening cost:

The manufacture performs 100% inspection when the items are produced from the production line. Thus, the screening cost is given by

\[
\text{Screening cost} = x Q
\]

(8)

Synthesizing the costs described in the above, we have the total cost per cycle shown as follows:

\[
TC_j(Q,K) = K + c_j Q + c_j f_g \left[ \frac{Q^2 \pi \cdot (1-p)}{2M} \right] + \left[ \frac{(p-p)Q^2}{2D} \right]
\]

\[
+ c_j f_d \left[ \frac{pQ^2 \pi}{2M} \right] + x Q + \frac{1}{\alpha} \left[ \ln \left( \frac{K_0}{K} \right) \right] \frac{Q (1-p)}{D}, \forall j = 1,2,\ldots, r
\]

The total profit, $TP_j(Q,K)$, per cycle is the total revenue less the total cost and is given as

\[
TP_j(Q,K) = TR_j(Q) - TC_j(Q,K)
\]

\[
= s Q (1-p) + v Q p - K - c_j f_g \left[ \frac{Q^2 \pi \cdot (1-p)}{2M} \right] + \left[ \frac{(p-p)Q^2}{2D} \right]
\]

\[
- p c_j f_d \left[ \frac{Q^2 \pi}{2M} \right] - x Q - \frac{1}{\alpha} \ln \left( \frac{K_0}{K} \right) \frac{Q (1-p)}{D} - c_j Q, \forall j = 1,2,\ldots, r
\]

Taking the expected value of the total profit per cycle $TP_j(Q,K)$ with respect to $p$, we have
Referring to Eq. (1) and taking the expected duration of cycle time $T$ with respect to $p$, we have

$$E_T = \frac{Q(1 - E[p])}{D}$$

Using the renewal-reward theorem (Ross, 1996), the expected profit per unit time can be expressed as follows:

$$E_T = \frac{Q(1 - E[p])}{D}$$

To maximize the expected total profit per unit time, we need to show that the Hessian Matrix of Eq. (9) is negatively definite. However, it is not easy to determine the concavity of the Hessian Matrix in Eq. (9). We therefore employ an alternative procedure to obtain the profit function properties.

### 4. Methodology and algorithm

To find the unique solution for the expected profit $E_T$, we need the following property

**Property 1:** For fixed $Q$, $E_T(Q, K)$ is concave in $K$

Employing Property 1, we obtain the possible optimal value of $K$ by letting $\partial E_T(Q, K)/\partial K = 0$, we have

$$\tilde{K}_j(Q) = \frac{E_j Q}{\alpha D}, \forall j = 1, 2, ..., r$$

Substituting Eq. (10) into Eq. (9), one has
This implies the feasible Q corresponding to each j
Q = \frac{1}{\alpha} \ln \left( \frac{\alpha D K_0}{E_i Q} \right)
\frac{Q c_j f_g}{E_i} \left\{ \frac{D \pi \cdot (1 - E[p])}{2M} + \frac{\pi^2 - 2E[p] \pi + E[p^2]}{2} \right\}
- \frac{Q c_j f_d}{E_i} \left[ \frac{D \pi E[p]}{2M} \right],
\forall j = 1, 2, ..., r.
(11)

Taking the first and second partial derivatives of Eq. (11) with respect to Q, we have
\frac{\partial ETPU_j(Q)}{\partial Q} = \frac{1}{\alpha Q} - \frac{c_j f_g}{E_i} \left\{ \frac{D \pi \cdot E_i}{2M} + \frac{\pi^2 - 2E[p] \pi + E[p^2]}{2} \right\}
- \frac{c_j f_d}{E_i} \left[ \frac{D \pi E[p]}{2M} \right],
\forall j = 1, 2, ..., r.
(12)
\frac{\partial^2 ETPU_j(Q)}{\partial Q^2} = -\frac{1}{\alpha Q^2} < 0 \ \forall j = 1, 2, ..., r.
(13)

Eqs. (12) and (13) reveal that the candidate optimal lot size \( Q_j^* \) corresponding to each procurement cost \( c_j \) exists and is unique. Letting \( \frac{\partial ETPU_j(Q)}{\partial Q} = 0 \), we obtain the possible optimal value of Q as follows:
\[ \tilde{Q}_j = \frac{E_i}{\alpha \left\{ c_j f_g \left[ \frac{D \pi E_i}{2M} + \frac{\pi^2 - 2E[p] \pi + E[p^2]}{2} \right] + c_j f_d \left[ \frac{D \pi E[p]}{2M} \right] \right\}}, \]
\forall j = 1, 2, ..., r.
(14)

We note that \( \tilde{Q}_j \) corresponding to the unit-purchasing cost \( c_j \) is valid of \( Q_{j-1} \leq \tilde{Q}_j < Q_j \).

However, \( \tilde{Q}_j \) may not exist at \( \left[ Q_{j-1}, Q_j \right) \) and thus two cases may occur:

**Case I:** \( \tilde{Q}_j > Q_j \), where \( \tilde{Q}_j \) is the maximum lot size corresponding to \( c_j \)

In this case, the buyer adopts the lower unit-procurement cost (say \( c_b \) and \( c_b < c_j \)) to obtain the lot size. From Eq. (11), we have \( ETPU_b(\tilde{Q}) > ETPU_j(\tilde{Q}) \). This implies the feasible candidate \( \tilde{Q}_j \) corresponding to \( c_j \) would not be the global optimization.

**Case II:** \( \tilde{Q}_j \leq Q_{j-1} \), where \( Q_{j-1} \) is the minimum lot size corresponding to \( c_j \)

In this case, we can fix \( Q \) and \( K \) respectively under given \( c_j \) and then two scenarios occur:

**Scenario I:** the perspective of the breakpoint \( Q_{j-1} \) corresponding to \( c_j \)

In this scenario, the candidate optimal lot size may occur at the break point \( Q_{j-1} \) with its corresponding candidate optimal setup cost. This is because the unit-procurement cost
depended on the lot size and thus has a different cost. Thus, for given \( Q_{j-1} \) corresponding to \( c_j \), the setup cost, similar to Eq. (10), could be determined as follows:

\[
\hat{K}_j = \frac{Q_{j-1}E_1}{aD}, \quad \forall j = 1, 2, ..., r. \tag{15}
\]

**Scenario II:** the perspective of fixed \( K \)

In this scenario, the candidate optimal solution may occur at \( \left( Q_j^*(K), K \right) \) satisfying \( Q_{j-1} \leq Q_j^* < Q_j \). Thus, given Eq. (9) for fixed \( K \), we have

\[
\frac{\partial ETPU_j(Q, K)}{\partial Q} = \frac{KD}{Q^2E_1} - \frac{c_jf_g}{E_1} \left( \frac{D\pi \cdot E_1 + \pi^2 - 2E[p]\pi + E[p^2]}{2M} \right)
\]

\[
- \frac{c_jf_g}{E_1} \left( \frac{D\pi E[p]}{2M} \right),
\]

\( \forall j = 1, 2, ..., r. \tag{16} \)

\[
\frac{\partial^2 ETPU_j(Q, K)}{\partial Q^2} = \frac{-2KD}{Q^3E_1} < 0 \tag{17} \]

Letting \( \frac{\partial ETPU_j(Q, K)}{\partial Q} = 0 \), we have

\[
Q_j^*(K) = \sqrt{\frac{2KD}{c_jf_g \left( \frac{D\pi E_1 + \pi^2 - 2E[p]\pi + E[p^2]}{M} \right) + c_jf_g \left( \frac{D\pi E[p]}{M} \right)}} ,
\]

\( \forall j = 1, 2, ..., r. \tag{18} \)

Plugging Eq. (18) into \( Q_{j-1} \leq Q_j^*(K) \), we have the following results

\[
K_j^* = \frac{\left( Q_{j-1} \right)^2 c_jf_g \left[ \frac{D\pi E_1}{M} + \pi^2 - 2E[p]\pi + E[p^2] \right] + c_jf_g \frac{D\pi E[p]}{M}}{2D}
\]

\( \forall j = 1, 2, ..., r. \tag{19} \)

Until now, the candidate optimal lot size and setup cost corresponding to unit-procurement cost has been obtained. To find the global optimal solution, an algorithm is developed as follows:

**Algorithm:**

Step1. Obtained \( \tilde{K}_j(Q) \) from Eq. (10) and \( \tilde{Q}_j \) from Eq. (14) for all \( j = 1, 2, ..., r. \)

Step2. For \( j = 1 \) to \( r \)

If \( \tilde{Q}_j > Q_j \), Next \( j \)

Else
If \( Q_{j-1} \leq \tilde{Q}_j < Q_j \) then Do {
    
    \{compute \( ETPU_j(\tilde{Q}_j, \tilde{K}_j) \) from Eq. (9) and record it\}

    \{Next \( j \)\}

} 

Else {

If \( \tilde{Q}_j < Q_{j-1} \), then Do {

\{obtain \( \tilde{K}_j \) from Eq. (15) and compute \( ETPU_j(Q_{j-1}, \tilde{K}_j) \) from Eq. (9)\}

\{obtain \( K^d_j \) from Eq. (19) and compute \( Q^d_j(K^d_j) \) from Eq. (18).\}

If \( Q_{j-1} \leq Q^d_j(K^d_j) < Q_j \) {

Compute \( ETPU_j(Q^d_j, K^d_j) \) from Eq. (9)

} 

Else {

\( ETPU_j(Q^d_j, K^d_j) = -\infty \)

} 

\{compare \( ETPU_j(\tilde{Q}_j, \tilde{K}_j) \), \( ETPU_j(Q_{j-1}, \tilde{K}_j) \), \( ETPU_j(Q^d_j, K^d_j) \)

and then record the maximum one\}

\{Next \( j \)\}

}

Step3. The maximum expected profit obtained in Step2 provides the optimal production lot size, setup cost, and the unit-procurement cost.

Note that if we ignore the capital investment in setup cost reduction, and quantity discounts and then consider all items are perfect quality (i.e. \( p=0 \) and \( h=c_jf_c = c_jf_d \)), the optimal lot size in Eq. (18) reduces to

\[
Q^* = \frac{2KD}{h(1-D/M)},
\]

which is equivalent to the result in the classical economic production quantity model. This helps for validating our model.

5. Numerical example

To illustrate the optimal lot size and setup cost \((Q^*, K^*)\) behavior, let us consider the following example.

Demand rate, \( D=2400 \) units/year,
Production rate, \( M=19200 \) units/year,
Original setup cost, \( K_0=$400/cycle,\)
Screening cost, \( d = $1/\text{unit} \)

Holding cost rate for a unit of good item (a fraction of dollar value) \( f_g = $0.05/\$\text{/year} \)

Holding cost rate for a unit of defect item (a fraction of dollar value) \( f_d = $0.025/\$\text{/year} \)

Selling price of good quality items, \( s = $50/\text{unit} \)

Salvage value of scrap items, \( v = $25/\text{unit} \)

In addition, the percentage scrap \( p \) is uniformly distributed over the range \([0, 0.04]\), then the probability density function \( f(p) \) is

\[
f(p) = \begin{cases} 
25, & 0 \leq p \leq 0.04 \\
0, & \text{otherwise} 
\end{cases}
\]

Furthermore, the supplier offers a price discount schedule as shown in Table 2.

<table>
<thead>
<tr>
<th>( j )</th>
<th>( Q_{j-1} \sim Q_j )</th>
<th>( c_j )</th>
</tr>
</thead>
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<td>( 0 &lt; Q &lt; 200 )</td>
<td>( c_1 = 20.05 )</td>
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<tr>
<td>2</td>
<td>( 200 \leq Q &lt; 500 )</td>
<td>( c_2 = 20.04 )</td>
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<tr>
<td>3</td>
<td>( 500 \leq Q &lt; 900 )</td>
<td>( c_3 = 20.03 )</td>
</tr>
<tr>
<td>4</td>
<td>( 900 \leq Q &lt; 1400 )</td>
<td>( c_4 = 20.02 )</td>
</tr>
<tr>
<td>5</td>
<td>( Q \geq 1400 )</td>
<td>( c_5 = 20.01 )</td>
</tr>
</tbody>
</table>

Considering the type of cost structure and using the algorithm developed in section 4, we obtain the optimal lot size, setup cost, and maximum expected total profit as follows:

**Step 1:** From Eq. (10) and Eq. (14), we have

\[
\begin{align*}
\tilde{Q}_1 &= 932.2, \quad \tilde{Q}_2 = 932.7, \quad \tilde{Q}_3 = 933.1, \quad \tilde{Q}_4 = 933.6, \quad \tilde{Q}_5 = 934.1 \\
\tilde{K}_1 &= 152.2, \quad \tilde{K}_2 = 152.3, \quad \tilde{K}_3 = 152.4, \quad \tilde{K}_4 = 152.5, \quad \tilde{K}_5 = 152.6
\end{align*}
\]

**Step 2:**

1. Because \( \tilde{Q}_1 > 200, \tilde{Q}_2 > 500, \tilde{Q}_3 > 900 \), this implies the optimal solution may not occur at these results. We therefore do not further compute them.

2. Because \( 900 \leq \tilde{Q}_4 < 1400 \), we substitute \( \tilde{Q}_4 = 933.6 \) and \( \tilde{K}_4 = 152.5 \) into Eq. (9) and thus obtain \( \text{ETPU}_4(933.6, 152.5) = 710099 \)

3. Because \( \tilde{Q}_5 < 1400 \), from Eq. (15), we then have

\[
\tilde{K}_5 = 152.6 \quad \text{and} \quad \text{ETPU}_5(1400, 152.6) = 709972 \]

4. Obtaining \( K^*_j \) from Eq. (19) and \( Q^*_j \) from Eq. (18), we have

\[
K^*_5 = 342.7 \quad \text{and} \quad Q^*_5 = 1400
\]

Substituting them into Eq. (9), one has \( \text{ETPU}_5(1400, 342.7) = 685106 \)
Step 3: Compare the results obtained in Step 2, we know $ETPU_4(933.6,152.5) = 710099$ gives the maximum expected total profit. This implies the optimal lot size is 933.6 units and the optimal setup cost is $152.5, which corresponding to the procurement cost $c_4=20.02$.

The optimal solution for the given parameter set is $Q^*=933.6$ units and $K^*=$$152.5, and the expected annual profit is $71009.9, which meets the results obtained in our developed Algorithm.

6. Conclusions

In this paper, we developed a new EPQ model with imperfect production processes and quantity discounts, where the screening process occurs at the production stage. An optimal set up cost reduction and lot sizing are the decision variables employing to construct an EPQ model. There were several different considerations included in this new model: (1) the different holding cost for good items and defective items (2) the quantity discounts (3) the capital investment (4) imperfect production processes. The expected total profit function was derived and a procedure associated with an algorithm was established to find the optimal solution. The work of the traditional EPQ model is a special case used in our model associated $c_{f,0}=c_{f,e}=h$ and $p=0$ when we ignore the capital investment in set up cost reduction and quantity discounts. Numerical results in general showed that: (1) the lowest unit purchasing cost in the cost discount schedule may not guarantee that the retailer could obtain maximum expected total profit because the extra quantity purchased may add additional holding cost and thus reduce the expected total profit; (2) the lower the holding cost rate for a unit of good item, the lower the optimal order quantity, set up cost and the expected annual total profit. However, if the optimal order quantity occurs at the ordering quantity break point corresponding to the least unit purchasing cost of the cost discount schedule, the optimal order quantity then remains unchanged with the holding cost rate for a unit of good items, while the set up cost increases and the expected total profit decreases; and (3) the higher the defective rate, the higher the order quantities and set up cost, and the less expected total profit. If the optimal order lot size occurs at the ordering quantity break point corresponding to the lowest cost in the cost discount schedule, the optimal lot size remains unchanged with the defective rate, while the set up cost increases and the expected total profit decreases.

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