Proceedings of the First European Academic Research Conference on Global Business, Economics, Finance and Social Sciences (EAR15Italy Conference) ISBN: 978-1-63415-028-6
Milan-Italy, June 30-July 1-2, 2015, Paper ID: I513

Measuring the Downside Risk of the Exchange-Traded Funds: Do the Volatility Estimators Matter?

Hung-Chun Liu,
Department of Finance,
Minghsin University of Science & Technology, Taiwan.
E-mail: hungchun65@gmail.com

Jying-Nan Wang,
Department of Applied Economics,
Fo Guang University, Taiwan.
E-mail: jyingnan@gmail.com

Yuan-Teng Hsu,
College of Management,
Yuan Ze University, Taiwan.
E-mail: yuanteng.hsu@gmail.com

Abstract

This paper aims to propose the augmented GJR-GARCH (GJR-GARCH$_M$) model which extends the GJR-GARCH model by including three volatility estimators, overnight volatility (ONV), daily prices range (PK), and implied volatility (VIX) as explanatory variables for the variance equations in GJR. The proposed value-at-risk (VaR) models are used to estimate the daily VaR values and evaluate their downside risk management performance for the SPDRs over the period from 2009 to 2014. Empirical results indicate that the GJR-GARCH$_M$ model outperforms the GJR-GARCH model for most cases, suggesting that the GJR-GARCH-based VaR forecasts can be moderately improved with the additional information contained in ONV, PK and VIX. In addition, daily prices range and implied volatility are far more informative than the overnight volatility estimator for improving the GJR-GARCH-based VaR forecasts. Market practitioners can adopt the proposed model for estimating and managing the potential loss of ETFs in the face of catastrophic events.

Key words: *Exchange-traded funds, SPDRs, Value-at-Risk, Volatility estimator, GJR*
JEL Classification: *C52; C53; G32*

1. Introduction

Exchange-traded funds (ETFs) are very popular and have become extensively adopted investment instruments among global investors over recent years. ETFs are attractive as investments because of their low expense ratios, tax efficiency, diversified-portfolio and stock-like features. In the early 1990s, the American Stock Exchange (AMEX) introduced Standard & Poor's Depositary Receipts (SPDRs, or Spider), which are backed by a stock portfolio that closely tracks the S&P 500 index. By far, the Spider is the most actively traded and the largest passive ETF worldwide, with US$215.91 billion under management as at January, 2015.

Researchers have long been aware that returns volatility changes over time and that period of high volatility tend to be found in clusters. The autoregressive conditional heteroskedastic (ARCH) model of Engle (1982) and the generalized autoregressive conditional heteroskedastic (GARCH) model advocated by Bollerslev (1986) respond to address these stylized phenomena. Since then, volatility forecasting technique has been dominated by a variety of the GARCH genre of models, especially for the asymmetric GARCH model. Glosten et al. (1993) propose the so-called GJR-GARCH model which is a simple class of GARCH-type models that can capture asymmetric effects of good news and bad news on conditional volatility.

Brooks et al. (2000) propose the overnight volatility (ONV) in order to capture accumulated overnight information which would be useful for capturing the persistence in the conditional heteroscedasticity of stock returns. Motivated by the daily price range, on the one hand, Parkinson (1980) uses the scaled high-low price ranges to develop the daily PK volatility estimator based on the assumption that intraday prices follow a Brownian motion process. On the other hand, Garman and Klass (1980) develop the GK estimator by using opening and closing prices in addition to price range, with assumptions similar to those of the PK estimator. In addition, Rogers and Satchell (1991) propose an estimator, RS, by including the drift in the price process. In 1993, Chicago Board Options Exchange (CBOE) proposes the implied volatility index (VIX) which is derived from the S&P 500 index option prices data with an option pricing model.

Recently, the broad availability of intraday trading data has inspired researchers to explore their information value in modeling and forecasting the volatility of financial markets (Blair et al., 2001; Koopman et al., 2005; Corrado and Truong, 2007; Vipul and Jacob, 2007; Fuertes et al., 2009). However, despite an extensive literature on volatility forecasting, none of them investigates the prices information which is embodied in the ONV, PK and VIX volatility estimators for improving predictive accuracy of daily value-at-risk forecasts in ETF. This study aims to propose the augmented GJR model which extends the traditional GJR-

GARCH model by including three volatility estimators, overnight volatility (ONV), daily prices range (PK), and implied volatility (VIX) as explanatory variables for the variance equations in GJR model. The proposed value-at-risk (VaR) models are used to estimate their daily VaR values and evaluate their downside risk management performance for the SPDRs returns spanning from 2009 to 2014.

The remainder of this study is organized as follows. The data and methodology are provided in Section 2, followed in Section 3 by the empirical results of daily VaR forecasts performance for SPDRs across alternative confidence levels. Conclusions drawn from this study are summarized in the final Section.

2. Methodology

2.1 Data and Preliminary Analysis

The data examined in this study comprises of the daily open, high, low, and closing prices data on SPDRs as well as the VIX data obtained from the Yahoo Finance website. The sample period for these daily data covers from 2 January 2009 to 31 December 2014 for a total of 1,510 trading days. The first four years (1,006 observations) are used as the in-sample period for estimation purpose, while the remaining two years (504 observations) are taken as the out-of-sample for forecast evaluation.

Table 1 provides the descriptive statistics of the daily returns for the Standard & Poor's Depositary Receipts. As showed in Table 1, the average daily return is positive, and approaches close to zero. The returns series exhibits significant evidence of skewness and kurtosis, which means that the series is skewed to the left, and the distribution of the daily returns is more fat-tailed and high-peaked than normal distribution. The J-B test statistic further confirms that the daily returns are non-normal distributed. Finally, the Ljung-Box test statistic displays linear dependence for the squared returns and strong ARCH effects.

Table 1: Descriptive statistics of daily returns for the SPDRs

This table presents the descriptive statistics of daily returns for the Standard & Poor's Depositary Receipts. J-B represents the statistics of Jarque and Bera (1987)'s normal distribution test. Q_s (12) refers to the Ljung-Box Q test statistic of the squared return series for up to the 12th order serial correlation. * indicates significance at the 1% level.

Mean (%)	Std.	Min	Max	Skew	Kurt	J-B	$Q_s(12)$
0.053	1.142	-6.734	6.960	-0.270*	4.484*	1283.019*	796.501*

2.2 Augmented GJR Model

We propose the augmented GJR model which extends the GJR-GARCH model of Glosten et al. (1993) by including various volatility estimators (ONV, PK and VIX), respectively, for its variance equation as follows:

$$R_t = \mu + \varepsilon_t\,,\ \varepsilon_t = \sigma_t z_t\,,\ z_t\,|_{\Omega_{t-1}} \sim NID(0,1) \tag{1}$$

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \gamma d_{t-1}\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \delta v_{t-1} \tag{2}$$

Where R_t is daily SPDRs return; μ denotes the conditional mean of returns; ε_t is the innovation process; z_t is the standardized residual with zero mean and unit variance; σ_t^2 is the conditional variance. d_{t-1} denotes the indicator function that takes the value of unity if $\varepsilon_{t-1} < 0$, and 0 otherwise. The indicator variable differentiates between positive (good news) and negative (bad news) shocks, so that asymmetric effects in the data are captured by γ. Thus, in the augmented GJR model, good news has an impact of α, and bad news has an impact of $(\alpha+\gamma)$, with bad (good) news having a greater effect on volatility if $\gamma > 0$ ($\gamma < 0$). Finally, v_{t-1} is a volatility estimator made at day $t-1$, including ONV (overnight volatility), PK (daily PK price range), and VIX (implied volatility). Table 2 provides a synopsis of these volatility estimators:

Table 2: The synopsis of various volatility estimators

This table presents the various volatility estimators employed in this study. O_t, H_t, L_t and C_t denote the opening, high, low, and closing prices at day t, respectively.

Abbreviation of volatility estimators	Studies	Formula or explanation	
ONV	Brooks et al. (2000)	$\hat{\sigma}_{ONV,t}^2 = (\ln(O_t / C_{t-1}))^2$	(3)
PK	Parkinson (1980)	$\hat{\sigma}_{\mathrm{PK,t}}^2 = (4\ln 2)^{-1} \cdot (\ln(\mathrm{H_t} / \mathrm{L_t}))^2$	(4)
VIX	-	VIX is a popular measure of the implied volatility of S&P 500 index options, which represents one measure of the market's expectation of stock market volatility over the next 30 day period. For consistent scaling with other daily volatility estimators, all VIX indexes are squared and divided by 252.	

2.3 Downside Risk Measurement and Performance Evaluation

The GJR-based VaR forecasts for a one-day holding period can be calculated as follows:

$$VaR_t = Z_{\alpha_1} \cdot \hat{\sigma}_t + \mu \tag{5}$$

Where Z_{α_1} denotes the corresponding quantile of the standard normal distribution at α_1, while $\hat{\sigma}_t$ is the volatility forecast generated from either GJR, GJR-ONV, GJR-PK or GJR-VIX model.

To backtest the VaR results, this study first employs a likelihood-ratio test by Kupiec (1995) to test whether the true failure rate is statistically consistent with the VaR model's theoretical failure rate. The null hypothesis of the failure rate *P* is tested against the alternative hypothesis that the failure rate is different from *P*, in which statistics is given by:

$$LR_{uc} = 2\ln\left[\frac{\hat{\pi}^{n_1}(1-\hat{\pi})^{n_0}}{P(1-P)^{n_0}}\right] \sim \chi^2(1) \tag{6}$$

Where $\hat{\pi} = n_1/(n_0 + n_1)$ is the maximum likelihood estimate of *P*, and n_1 denotes a Bernoulli random variable representing the total number of VaR violations.[1]

Christoffersen (1998) developed a conditional coverage test (LR_{cc}) that jointly investigates whether the total number of failures is equal to the expected one, and the VaR violations are independently distributed. Given the realizations of the SPDRs returns series R_t and the set of VaR estimates, the indicator variable I_t can be defined as follows:

$$I_t = \begin{cases} 1 & \text{,if } R_t < VaR_t \\ 0 & \text{,if } R_t \geq VaR_t \end{cases} \tag{7}$$

Since accurate VaR estimates display the property of correct conditional coverage, the I_t series must exhibit both correct unconditional coverage and serial independence. The LR_{cc} test is a joint test of these two properties, and the corresponding test statistics is $LR_{cc} = LR_{uc} + LR_{ind}$ as we condition on the first observation. Consequently, under the null hypothesis that the failure process is independent and the expected proportion of violations equals *P*, the appropriate likelihood ratio is represented as follows:

$$\mathrm{LR}_{cc} = -2\ln\frac{(1-\mathrm{P})^{n_0}\,\mathrm{P}^{n_1}}{(1-\hat{\pi}_{01})^{n_{00}}\,\hat{\pi}_{01}^{n_{01}}\,(1-\hat{\pi}_{11})^{n_{10}}\,\hat{\pi}_{11}^{n_{11}}} \sim \chi^2(2) \tag{8}$$

Where $n_{i,\,j}$ = the number of observations with value *i* followed by value *j* ($i, j = 0, 1$), $\pi_{ij} = P\{I_t = j \mid I_{t-1} = i\}\ (i, j = 0,1)$, $\hat{\pi}_{01} = n_{01}/(n_{00} + n_{01})$, $\hat{\pi}_{11} = n_{11}/(n_{10} + n_{11})$.

3. Empirical results and analysis

3.1 Empirical analysis

Table 3 presents out-of-sample daily VaR forecasts performance across the various models by reporting mean VaR, violation, failure prob., LR_{uc} and LR_{cc} statistics, under 90%, 95% and 99% confidence levels.

As shown in Table 3, the GJR model generates the highest average absolute VaR estimates at every confidence level, and followed by the GJR-ONV, GJR-PK and GJR-VIX models. Thus, the GJR and the GJR-VIX models generate the lowest and highest numbers of VaR violations, respectively.

Panel A of Table 3 provides daily VaR forecasts results for SPDRs at the 90% confidence level. We observe that either the GJR or the GJR-ONV model fails to pass the unconditional coverage test (LR_{uc}), indicating that both traditional GJR and GJR-ONV models tend to over-predict VaR values for SPDRs returns. Moreover, the GJR model has been rejected by the conditional coverage test (LR_{cc}), indicating that clustered violations were generated. That is,

[1] If the predicted VaR cannot cover the realized dollar loss, this is termed as a violation.

the GJR model is very slow at updating the VaR value when market volatility changes rapidly. By contrast, both the GJR-PK and the GJR-VIX models pass the coverage tests, suggesting that the empirical failure probability is statistically consistent with the prescribed one for each of them, especially for the latter model. Meanwhile, with any sudden change in market volatility, the GJR-PK and the GJR-VIX models are beneficial for rapidly updating the VaR value. Thus, the trading prices information which is implied in PK and VIX volatility measures is crucial for producing adequate daily VaR forecasts for SPDRs returns at the 90% confidence level.

For the case of 95% confidence level, we observe that the LR_{uc} test statistic is insignificant for the GJR, GJR-ONV and GJR-PK models, indicating that the sample point estimate is statistically consistent with the prescribed confidence level of these three VaR models. The LR_{cc} statistic further shows that the aforesaid three models also can pass the conditional coverage test, indicating that these models' performance is quite stable over time during the out-of-sample forecasting period 2013~2014. However, the GJR-VIX model fails to offer adequate VaR forecasts according to the LR_{uc} test statistic.

The VaR forecasts results at the 99% confidence level are very similar to those obtained at the 95% confidence level. That is, the LR_{uc} and LR_{cc} statistics reported in Panel C of Table 3 are all insignificant, except for the GJR-VIX model, indicating that the GJR, GJR-ONV and GJR-PK models are able to produce adequate Daily VaR forecasts for SPDRs returns.

Table 3: Daily VaR forecasts results

This table presents daily VaR forecasts results for SPDRs at three confidence levels. The critical values of the LR_{uc} and LR_{cc} statistics at the 10% significance level are 2.71 and 4.61, respectively. Figures in bold text indicate rejection of the null hypothesis of correct VaR estimates at the 10% significance level.

Model	Mean VaR	Violation	Failure prob.	LR_{uc}	LR_{cc}
Panel A: 90% Confidence Level					
GJR	-1.0192	36	7.14%	**5.02**	**5.15**
GJR-ONV	-0.9864	37	7.34%	**4.32**	4.51
GJR-PK	-0.9142	42	8.33%	1.63	2.52
GJR-VIX	-0.8268	53	10.51%	0.14	0.63
Panel B: 95% Confidence Level					
GJR	-1.3166	25	4.96%	0.00	0.05
GJR-ONV	-1.2744	26	5.15%	0.02	0.10
GJR-PK	-1.1838	26	5.15%	0.02	0.19
GJR-VIX	-1.0701	34	6.74%	**2.92**	2.95
Panel C: 99% Confidence Level					
GJR	-1.8745	5	0.99%	0.00	3.73
GJR-ONV	-1.8146	5	0.99%	0.00	3.73
GJR-PK	-1.6897	7	1.38%	0.68	3.27
GJR-VIX	-1.5265	11	2.18%	**5.32**	**6.48**

4. Conclusions

This study proposes the augmented GJR model which extends the GJR-GARCH model of Glosten et al. (1993) by including overnight volatility, daily prices range, and implied volatility as explanatory variables for the variance equations in the GJR model. These VaR models are used to estimate their daily VaR values and evaluate their downside risk management performance for the SPDRs returns covering from 2009 to 2014. Empirical results indicate that the augmented GJR models outperform the GJR model for most cases, suggesting that the GJR-GARCH-based VaR forecasts can be moderately improved with the additional information contained in ONV, PK and VIX volatility estimators. In addition, daily prices range and implied volatility are far more informative than the overnight volatility for improving the GJR-GARCH-based VaR forecasts. Market practitioners can adopt the proposed model for estimating and managing the potential loss of ETFs in the face of catastrophic events.

References

Blair, B.J., Poon, S.H. and Taylor, S.J., 2001, Forecasting S&P 100 volatility: the incremental information content of implied volatilities and high frequency returns. *Journal of Econometrics* 105, 5 -26.

Bollerslev, T., 1986, Generalized autoregressive heteroskedasticity. *Journal of Econometrics* 31(3), 307–327.

Brooks, C., Clare, A.D. and Persand, G., 2000, A word of caution on calculating market-based minimum capital risk requirements. *Journal of Banking and Finance* 24, 1557-1574.

Christoffersen, P.F., 1998, Evaluating interval forecasts. *International Economic Review* 39, 841-862.

Corrado, C. and Truong, C., 2007, Forecasting stock index volatility: comparing implied volatility and the intraday high-low price range. *Journal of Financial Research* 30(2), 201-215.

Engle, R. F., 1982, Autoregressive conditional heteroscedasticity with estimates of the variance of U.K. inflation. *Econometrica* 50(4), 987–1008.

Fuertes, Ana-Maria, Izzeldin, M. and Kalotychou, E., 2009, On forecasting daily stock volatility: The role of intraday information and market conditions. *International Journal of Forecasting* 25(2), 259-281.

Garman, M. and Klass, M. 1980, On the estimation of security price volatilities from historical data. *Journal of Business* 53(1), 67-78.

Glosten, L., Jagannathan, R., Runkle, D., 1993, On the relation between the expected value and the volatility nominal excess return on stocks. *Journal of Finance* 46, 1779-1801.

Jarque, C.M. and Bera, A.K., 1987, A test for normality of observations and regression residuals. *International Statistics Review* 55, 163-172.

Koopman, S., Jungbacker, B. and Hol, E., 2005, Forecasting daily variability of the S&P 100 stock index using historical, realised and implied volatility measurements. *Journal of Empirical Finance* 12(3), 445-475.

Kupiec, P., 1995, Techniques for verifying the accuracy of risk measurement models. *Journal of Derivatives* 3, 73-84.

Parkinson, M., 1980, The extreme value method for estimating the variance of the rate of return. *Journal of Business* 53(1), 61-65.

Rogers, L.C.G. and Satchell, S.E., 1991. Estimating variance from high, low and closing prices. *Annals of Applied Probability* 1(4), 504-512.

Vipul and Jacob, J., 2007, forecasting performance of extreme-value volatility estimators. *Journal of Futures Markets* 27(11), 1085-1105.