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Idiosyncratic Risk and the Cross-Section of Expected Stock Return: A Threshold Regression Approach

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Abstract

The objective of this research is to re-examine the relationship between idiosyncratic risk and the cross-section of expected stock returns using threshold regression methods. Idiosyncratic risk should not bear a relationship with expected stock returns because it can be diversified away. Although previous researchers show enormous efforts in this regard, no consistent conclusion has been achieved. This paper employs threshold regression to uncover the underlying relationship between idiosyncratic risk and the cross-section of expected stock returns. Grounded on Merton (1987) that a positive relation exists between idiosyncratic risk and stock returns because investors are not well-diversified, we hypothesize that investors' incentive to diversify varies over time. Therefore, a positive relation exists when investors are less inclined to diversify, and a weaker or negative relation exists when investors have strong incentive to diversify. A threshold regression tests for the threshold(s) that such distinction exists. The results will shed light on the existing literature and help reconcile the conflicting results found in the literature.

Keywords: Stock returns; Idiosyncratic risk; Market efficiency; Investment decision.

1. Introduction

The risk in stock investments includes systematic risk and idiosyncratic risk. This paper focuses on idiosyncratic risk which is firm specific and has no correlation to the overall market risk. Modern portfolio theory indicates that the investors can hold a portfolio of stocks to diversify the idiosyncratic risk, hence idiosyncratic risk should not be compensated for higher returns in equilibrium. For various reasons, in particular a market with incomplete information, investors in reality may not hold perfectly diversified portfolio. In a less-than-perfect market, therefore, idiosyncratic risk may be compensated for higher returns. (Merton, 1987)

Some researchers predict that idiosyncratic volatilities will have a positive effect on the expected returns due to under-diversification, (see Malkiel and Xu, 2002, Levy, 1978, and Merton, 1987). Barberis and Huang (2001) predict that the higher idiosyncratic volatility should earn higher expected returns. Goyal and Santa-Clara (2003) also find a significant positive relation between average stock variance (largely idiosyncratic) and the value-weighted portfolio return on the NYSE/AMEX/Nasdaq stock for the period of 1963:08 to 1999:12. They postulate that under-diversified inventors demand a return compensation for bearing idiosyncratic volatilities. However, Bali, Cakici, Yan and Zhang (2005) find the result of Goyal and Santa-Clarea do not exist for the 1963:08 to 2001:12, and they also find there are no significant relation between the equal-weighted average stock volatility and the value-weighted portfolio return on the NYSE/AMEX or NYSE stocks. Bali and Cakici (2008) also find that no robustly significant relation exists between volatility and expected returns.

On the other hand, Ang, Hodrick, Xing, and Zhang (2006) find that, in the cross-section of stocks, high idiosyncratic volatility in one month predicts abysmally low average returns in the next month. They find that the stocks with high sensitivities to innovations in aggregate volatility have low average return and firms with high idiosyncratic volatility have very low average return. They think time-varying market volatility induces changes in the investment opportunity set by changing the risk-return trade-off. Nonetheless, Fu (2009) refutes this negative relationship as he finds a significantly positive relation between the estimated conditional idiosyncratic volatilities and expected returns. He employs the exponential generalized autoregressive conditional heteroscedasticity (EGARCH) model and out-of-sample data to estimate expected idiosyncratic risk and then run Fama-MacBeth regressions of monthly stock returns and other firm characteristics that are known to explain cross-sectional returns. The positive relation is both statistically and economically significant. Guo et al. (2014), however, argue that the look-ahead bias is problematic and the empirical idiosyncratic risk-return relation becomes negligible when the look-ahead bias is corrected. Huang et al. (2010) argue that the negative relation between idiosyncratic risk and stock returns disappears after return reversals are controlled for. However, a positive relation still exists in the monthly data. Boehme et al (2009) find evidence supporting Merton (1987) that stocks with low levels of investor recognition and for which short selling is limited, the relation between idiosyncratic risk and stock returns is positive. Vozlyublennaia (2012) analyzes the relationship using a GARCH-in-Mean framework, and find 15% of stocks exhibit a significant relationship between returns and risk, of which 9% are positive. Moreover, these proportions vary over time and with model specifications. Guo and Qiu (2014) test the idiosyncratic risk-return relation using options-implied volatility, a forward looking measure of conditional variance. They find that the negative relation gets stronger when short-sale constraints become more prevalent. Finally, Eiling (2013) argue that human capital is an important asset pricing factor. Idiosyncratic risk may appear to be priced when human capital is excluded from the model.

Although previous researchers have committed efforts in resolving these controversies; however, no consistent conclusion has been achieved. This paper will re-examine this issue because we postulate that investors' incentive to diversify their portfolios depends on the market sentiment. Previous researchers did not consider the fact that time-varying market cycles will affect investors' investment preference, risk aversion, and diversification incentives. For example, when investors are momentum traders in a bull market; they might be poorly diversified. The frenzy trading of high tech stocks during the internet bubble periods is a clear example. Such market imperfection may result in a significant relation between idiosyncratic risk and stock returns due to the lack of diversification incentives. On the other hand, during the bear markets, investors turn risk-averse, hence may be better diversified. Investors' incentive to diversify between market cycles, therefore, should have differential impact on the relation between idiosyncratic risk and stock returns.

Given the fact that academic interest in this subject matter is strong and the conclusions are far from conclusive, we re-examine this issue from a different perspective and methodology. We propose a new methodology to uncover the underlying relationship between idiosyncratic risk and the cross-section of expected stock price returns. A threshold regression is appropriate for this purpose because we posit that the relation between idiosyncratic risk and stock returns is conditioned on the market conditions and the relation is evident only if the idiosyncratic risk is below a threshold, hence the objective of this research is to resolve these controversies using threshold regression methods.

2. Assumptions and Variables

Since we argue that investors' incentive to diversify varies across market conditions, the relation between idiosyncratic risk and expected stock returns cannot be time in-varying over a long period of time when the market experiences ups and downs. If Merton (1987) is correct, i.e., a positive relation between idiosyncratic risk and stock returns exists because investors do not always diversify, then we would expect to see such relation holds better during periods that investors turn aggressive as they are less inclined to diversify when they are over-confident. For example, during the latter part of the 1990s, investors aggressively chased internet stocks and were poorly diversified. On the other hands, after the market crashed, investors turned conservative and were better diversified. We thus argue that, in general, the relation between idiosyncratic risk and stock returns should be conditioned on the investors' inclination to diversify.

2.1 Assumptions and Hypotheses

Investors' desire to diversify, however, is difficult to measure. Therefore, some proxies must be used to capture such variable. We propose the Idiosyncratic risk as proxies, because in the bull market, investors' confident is high, so is idiosyncratic risk. Since investors turn aggressive, over-confident, and are less concerned about diversification, we argue that investors are less diversified when the idiosyncratic risk is high. Investors are less inclined to hold diversified portfolio, hence a positive relation between idiosyncratic risk and stock return is observed. On the other hand, in a bear market, stocks are more correlated with the market, thus investors incentive to diversify. hence higher (lower) systematic (idiosyncratic risk).

Based upon above arguments, we posit the following hypotheses.

Hypothesis : there is a threshold value of idiosyncratic risk such that the positive relation between idiosyncratic risk and stock returns is significantly positive when the idiosyncratic risk is above the threshold(s). Alternatively, the relation between idiosyncratic risk and stock returns is weak or insignificant when the idiosyncratic risk is below the thresholds.

2.2 Data Source and Variable Definition

The sample of data includes stocks traded on NYSE and Nasdaq during the period from January 1998 to December 2012. All data for the first stage estimates will be obtained from the CRSP database, Federal Reserve Bank of St. Louis, and French-Fama website. Variables for the second stage estimates are obtained from the Compustat database. The measurements of key variables *constructed for the measurement of the threshold variable* idiosyncratic volatility obtained by finding the standard deviation of the residuals from Fama-French 3-factor model. *Variables used and obtained from the first stage estimates* are explained below:

(1)Stock Returns (R_{it}): measured by the natural logarithm of the price ratio;

(2)Market Returns (R_{mt}): measured by the CRSP value-weighted returns;

(3)Risk-free Rate (R_{rf}): measured by three-month T-bill rate obtained from the Federal Reserve Bank of St. Louis;

(4)SMB: Fama-French's small minus big risk factor;

(5)HML: Fama-French's high minus low risk factor;

Other control variables include of Ln(ME): natural logarithm on the market value of equity; Ln(BE/ME): natural logarithm of BE over ME, where ME is the market value of equity and BE is the book value of equity; Ln(LEV): natural logarithm of financial leverage, measured by the debt to asset ratio.

3. Models

To test our hypotheses, we first estimate the idiosyncratic risk using the Fama-French 3-factor model, i.e.,

$$R_{it} - R_{ft} = \beta_0 + \beta_1(R_{mt} - R_{ft}) + \beta_2(SMB)_t + \beta_3(HML)_t + \varepsilon_{it} \tag{1}$$

The idiosyncratic volatility of stock is computed as the standard deviation of the regression residuals, i.e., $\sqrt{Var(\varepsilon_{it})}$. To reduce the impact of infrequent trading on idiosyncratic volatility estimates, we require a minimum of 15 trading days in a month for which CRSP reports both a daily return and non-zero trading volume.

Secondly, we construct a threshold regression with the expected stock return as the endogenous variable and idiosyncratic volatility as one of the explanatory variables to test the relationship between idiosyncratic risk and the cross-section of expected stock returns. Threshold regression methods are developed for non-dynamic panels with individual special fixed effects. It will be used to test the relation between idiosyncratic risks and stock returns. The regression model can be specified as:

{ $\boldsymbol{R_{it}, q_{it}, x_{it} : 1 \le i \le n, 1 \le t \le}$ T}. the subscript *i* indexes the individual stocks and the subscript *t* indexes time. The threshold variable q_{it} is a scalar and the thresholds are ordered so that $\gamma_1 < \gamma_2$ in the case that the number of thresholds is more than one. The threshold variable is idiosyncratic risk. We will conduct test described below to determine the number of thresholds.

$$\begin{aligned} R_{it} &= \beta'_{0t} + \beta'_{1t}[IVOL_{it}]I(q_{it} \le \gamma_1) + \beta'_{2t}[IVOL_{it}]I(q_{it} \le \gamma_2) \\ &+ \beta'_{3t}[IVOL_{it}]I(\gamma_1 \le q_{it} \le \gamma_2) + \beta'_{kt}\sum_{K=4}^{K} X_{kit} + u_{it} \end{aligned} \tag{2}$$

In Equation (2), R_{it} is the stock return, $IVOL_{it}$ is the idiosyncratic risk, and X_{it} are the explanatory variables including the threshold variables. $I(\cdot)$ is the indication function, and γ is the threshold value. Finally, u_{it} is the unobserved scalar random variable (errors).

The hypothesis of no threshold effect in (2) can be represented by the linear constraint

$$H_0: \ \beta_1 = \beta_2$$

Under the null hypothesis of no threshold, the model is

$$R_{it} = \mu_i + \beta_1^{'} x_{it} + e_{it}$$

After the fixed-effect transformation is made, we obtain

$$R_{it}^{*} = \beta_1^{'} x_{it}^{*} + e_{it}^{*}$$

The regression parameter β_1 is estimated by ordinary least squares (OLS), yielding estimated $\tilde{\beta}_1$, residuals $\tilde{e}_{it}^{*}$ and sum of squared errors $S_0 = \tilde{e}_{it}^{*'} \tilde{e}_{it}^{*}$. The likelihood ratio test of H_0 is based on

$$F_1 = (S_0 - S_1(\hat{\gamma})) / \hat{\sigma}^2$$

If F_1 rejects the null of no threshold, we need to further test to discriminate between one and two thresholds.

$$\hat{\sigma}^2 = S_2^r(\gamma_2^r) / n(T-1); \ F_2 = \frac{S_1(\hat{\gamma}_1) - S(\hat{\gamma}_2^r)}{\hat{\sigma}^2}.$$

The hypothesis of one threshold is rejected in favor of two thresholds if F_2 is large.

The asymptotic (1-∂)% confidence intervals for γ_2 and γ_1 are the set of values of γ such that $LR_2^r(\gamma) \le C(\partial)$ and $LR_1^r(\gamma) \le C(\partial)$, respectively, where Asymptotic (1-∂)% confidence intervals for γ_2 and γ_1 are the set of values of γ such that $LR_2^r(\gamma) \le C(\partial)$ and $LR_1^r(\gamma) \le C(\partial)$, respectively, where LR_2^{γ} and LR_1^{γ} are defined as:

$$LR_2^r = \frac{S_2^r(\gamma) - S_2^r(\hat{\gamma}_2^r)}{\hat{\sigma}^2}, \text{ and } \ LR_1^r = \frac{S_1^r(\gamma) - S_1^r(\hat{\gamma}_1^r)}{\hat{\sigma}^2}.$$

This research re-examine the relationship between idiosyncratic risk and the cross-section of expected stock price returns. We thus postulate that investors' incentive to diversify their portfolios depends on the market sentiment.

4. Empirical Results

When the market is pessimistic, high idiosyncratic risk stocks are more often associated with high financial distress firms which tend to earn low returns. Moreover, the high idiosyncratic risk limits the activity of arbitrageurs and short sellers, which discourages correcting potential mispricing. Therefore, idiosyncratic risk is negatively related to the stock return. However, when the market is in a more normal state, investors has less incentive to diversify and thus require risk premium to compensate for idiosyncratic risk, hence a positive relationship between idiosyncratic risk and stock returns. Such relation is more consistent with Merton's prediction [1987]. Merton shows that idiosyncratic risk plays a role in equilibrium, because investors cannot hold a perfect portfolio to diversify the idiosyncratic risk due to the incomplete information. Consequently, they demand compensation for securities' idiosyncratic risk. Thus, the cross-section stock return is positively related to its idiosyncratic risk. Our research thus provides an alternative explanation as to why the relationship between idiosyncratic risk and stock returns may be positive or negative.

Because we use the balanced panel data, for further re-examine the threshold effect, we delete the periods of variables with incomplete data and the variables with default value, and then we get the data from 1998 to 2012 years. We divide the data into 3 periods, each period has 5 years, and we test the threshold value again.

Table 1: The result of threshold regression

Period	**1998-2012**	**1998-2002**	**2003-2007**	**2008-2012**
Threshold number	3	3	3	2
IVOL Interval				
1	-	+	-	+
2	+	+	-	+
3	+	+	-	+
4	+	+	+	.

Ps:. Stands for no value.

There are 14220 sample numbers from 1998 to 2012, and 4740 in each 5-year period. The descriptive statistics of variables is on Table1 in Appendix A.The stock return is a dependent variable; the idiosyncratic risk is a threshold variable. The market value, book value over market value of equity and financial leverage are control variables. There are three threshold values from 1998-2012, 1998-2002 and 2003-2007 and two threshold values from 2008-2012, the threshold regression is on Table 1 in Appendix B. It shows that the idiosyncratic risk has different impact on stock returns in each period.

Appendix B shows the result of threshold regression. We test the period of 1998-2012; 1998-2002; 2003-2007; 2008-2012 years and there is significant threshold effect in each period to make the slope of idiosyncratic risk and expected stock return change. In the OLS model (see Table 2 in Appendix A), the bear/bull market effect is diluted due to multiple market cycles over a long period so the relationship of idiosyncratic risk and expected stock return is not significant. However, using threshold regression, we can observe a transition from negative to positive relationship between idiosyncratic risk and expected stock return with threshold value of idiosyncratic risk of 1998-2012 years. If we further divide the period into several 5-year periods, more significant threshold effect can be observed with more threshold values. We can find that the relationship between idiosyncratic risk and expected stock return is positive in 1998-2002 and 2008-2012, and the slope transits from negative to positive in 2003-2007 only. So the relationship between idiosyncratic risk and expected stock returns are often positive.

5. Conclusions

The objective of this research is to re-examine the relationship between idiosyncratic risk and expected stock returns using panel threshold regression methods in USA stock market. According to CAPM, idiosyncratic risk should not bear a relationship with expected stock returns because it can be diversified away. This paper employs threshold regression model to uncover the underlying relationship between idiosyncratic risks and expected stock returns in USA stock market. Grounded on Merton (1987) that a positive relation exists between idiosyncratic risk and stock returns because investors are not well-diversified, we hypothesize that investors' incentive to diversify varies over time. When investors have strong incentive to diversify during bear market, a weaker or negative relation exists.

The results show that the threshold regression can discriminate several intervals, where the relationship between idiosyncratic risk and expected stock returns changes with idiosyncratic risk. We use the USA stock market data with threshold regression to show that there is nonlinear relationship between idiosyncratic risks and stock returns. Our results support Merton (1987) argument, the relationship between idiosyncratic risk and expected stock returns are often positive. Investors are not well-diversified and the behavior of investors changes with the time and market economic.

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Appendix A

Table 1: Descriptive statistics of variables

Variable	Statistics	1998~2002	2003~2007	2008~2012	1998~2012
Ret	Mean	0.09	0.21	0.13	0.14
	Std.Dev	1.31	0.78	1.08	1.08
	Minimum	-0.96	-0.87	-0.95	-0.96
	Medium	-0.05	0.10	0.04	0.03
	Maximum	52.98	23.96	37.42	52.98
Lnme	Mean	6.22	6.89	6.85	6.65
	Std.Dev	2.17	2.01	2.13	2.13
	Minimum	-0.11	0.11	0.08	-0.11
	Medium	6.16	6.84	6.87	6.64
	Maximum	13.14	13.13	12.90	13.14
Lnbeme	Mean	-0.71	-0.88	-0.54	-0.71
	Std.Dev	0.82	0.65	0.82	0.01
	Minimum	-4.91	-6.35	-8.05	-8.05
	Medium	-0.64	-0.81	-0.47	-0.66
	Maximum	3.35	2.99-	2.65	3.35
Lnda	Mean	-0.75	-0.76	-0.74	-0.75
	Std.Dev	0.68	0.65	0.66	0.66
	Minimum	-6.63	-5.96	-8.42	-8.42
	Medium	-0.57	-0.61	-0.58	-0.59
	Maximum	-0.02	-0.04	0.00	0.00
IVOL	Mean	12.14	8.52	9.78	10.15
	Std.Dev	8.38	7.44	7.34	7.88
	Minimum	1.09	0.71	1.12	0.71

	Medium	9.80	6.75	8.07	8.15
	Maximum	143.44	221.21	173.46	221.21
beta	Mean	-1.82	10.31	5.31	4.60
	Std.Dev	29.69	27.06	36.37	31.68
	Minimum	-206.57	-198.17	-594.18	-594.18
	Medium	-4.10	3.64	4.84	2.10
	Maximum	191.70	418.20	243.25	418.20

Note: we obtain idiosyncratic risk [IVOL] from Fama-French 3-factor model. The sample period is Jan 1998 to Dec. 2012. The overall sample includes 14220 firms. Ret is monthly stock return. ME and BE are the calendar quarter-end market value of equity and book value of equity. leverage is the quarterly data.

Table 2: The OLS model from 1998-2012

	coefficient	std.error	t-value	p-value	
const	-0.2830	0.0400	-7.0800	0.0000	***
IVOL	0.0267	0.0012	21.9200	0.0000	***
ln_me	0.0056	0.0049	1.1320	0.2579	
ln_beme	-0.1668	0.0125	-13.3400	0.0000	***
ln_da	0.0254	0.0139	1.8280	0.0676	*
beta	0.0040	0.0003	14.4200	0.0000	***

Ps. *10% **5% ***1% significant

Table 3: The nonlinear testing from 1998-2012

	coefficient	std.error	t-value	p-value	
const	-0.2832	0.0778	-3.6400	0.0003	***
IVOL	0.0104	0.0019	5.5810	0.0000	***
ln_me	0.0624	0.0212	2.9500	0.0032	***
ln_beme	-0.0825	0.0188	-4.3850	0.0000	***
ln_da	0.0134	0.0266	0.5048	0.6137	
beta	0.0000	0.0003	0.0441	0.9648	
sq_IVOL	-0.0002	0.0000	-7.4840	0.0000	***
sq_ln_me	-0.0046	0.0015	-3.1090	0.0019	***
sq_ln_beme	-0.0367	0.0058	-6.3130	0.0000	***
sq_ln_da	0.0007	0.0073	0.1004	0.9200	
sq_beta	0.0000	0.0000	2.2630	0.0236	**

Ps. *10% **5% ***1% significant

Appendix B

Table 1: Threshold Regression Model of Each Period

Period	Threshold value	Threshold regression model
1998-2012	10.7468 14.4508 18.6302	$R_{it} = u_{it} - 0.0103 IVOL_{it} I(q_{it} \le 10.7468) + 0.0005 IVOL_{it} I(10.7468 < q_{it} \le 14.4508)$ $+0.0088 IVOL_{it} I(14.4508 \le q_{it} \le 18.6302) + 0.0158 IVOL_{it} I(q_{it} > 18.6302)$ $+0.0125 Lnme_{it} - 0.1591 Lnbeme_{it} - 0.0265 Lnda_{it} + 0.0001 Beta_{it} + e_{it}$
1998-2002	15.1754; 20.0446; 23.7586.	$R_{it} = u_{it} + 0.0073 IVOL_{it} I(q_{it} \le 15.1754) + 0.0223 IVOL_{it} I(15.1754 < q_{it} \le 20.0446)$ $+0.0092 IVOL_{it} I(20.0446 \le q_{it} \le 23.7586) + 0.0309 IVOL_{it} I(q_{it} > 23.7586)$ $+0.1941 Lnme_{it} - 0.7909 Lnbeme_{it} - 0.1131 Lnda_{it} - 0.0016 Beta_{it} + e_{it}$
2003-2007	6.7252; 12.6333; 16.0593.	$R_{it} = u_{it} - 0.0328 IVOL_{it} I(q_{it} \le 6.7252) - 0.018 IVOL_{it} I(6.7252 < q_{it} \le 12.6333)$ $-0.0015 IVOL_{it} I(12.6333 < q_{it} \le 16.0593) + 0.0228 IVOL_{it} I(q_{it} > 16.0593)$ $-0.1746 Lnme_{it} - 0.7488 Lnbeme_{it} - 0.4042 Lnda_{it} + 0.0043 Beta_{it} + e_{it}$

2008-2012	8.2088; 14.5425; 20.0419.	$R_{it} = u_{it} + 0.0104 IVOL_{it} I(q_{it} \le 14.5425)$ $+0.0341 IVOL_{it} I(14.5425 \le q_{it} \le 20.0419) + 0.0211 IVOL_{it} I(q_{it} > 20.0419)$ $+0.4134 Lnme_{it} - 0.505 Lnbeme_{it} - 0.0247 Lnda_{it} + 0.0036 Beta_{it} + e_{it}$

Table 2: Threshold Effect Testing (1998-2012)

	Single threshold effect testing	Double threshold effect testing	Triple threshold effect testing
Threshold value	14.4507513	10.74676037	18.63018417
F value	22.1731	6.6722	6.5008
P value	0.0000***	0.0100***	0.0200**
The critical value of F			
10%	2.754509172	2.432440183	2.658289357
5%	4.022526937	3.129477308	4.265707727
1%	7.572028971	6.693894673	8.92964711

註: *代表 10% **代表 5% ***代表 1% 之顯著水準

Table 3: Threshold Parameters Estimation (1998-2012)

Regressor	Coef	Std	t	prob
Lnme	0.0125	0.0179	0.6982	0.4851
Lnbeme	-0.1591	0.0241	-6.5942	0
Lnda	-0.0265	0.039	-0.6813	0.4957
Beta	0.0001	0.0005	0.2355	0.8139
IVOL(qi≤10.74676037)	-0.0103	0.0057	-1.8194	0.0689
IVOL(10.74676037<qi≤14.4507513)	0.0005	0.0042	0.1082	0.9138
IVOL(14.4507513<qi≤18.63018417)	0.0088	0.0036	2.4502	0.0143
IVOL(qi>18.63018417)	0.0158	0.0021	7.464	0

Table 4: Threshold Effect Testing (1998-2002)

	Single threshold effect testing	**Double threshold effect testing**	**Triple threshold effect testing**
Threshold value	23.7586	15.1754	20.0446
F value	12.4371	5.1083	5.4185
P value	0.0000***	0.0333**	0.0167**
The critical value of F			
10%	2.879566548	2.870459694	2.445885842
5%	3.442971933	4.260158929	3.373690524
1%	6.442355584	8.46407879	7.602593205

Ps. *10% **代表 5% ***1% significant

Table 5: Threshold Parameters Estimation (1998-2002)

Regressor	**Coef**	**Std**	**t**	**prob**
Lnme	0.1941	0.062	3.1309	0.0018
Lnbeme	-0.7909	0.071	-11.1321	0
Lnda	-0.1131	0.0913	-1.2391	0.2154
Beta	-0.0016	0.0007	-2.2698	0.0233
IVOL(qi≤15.1754)	0.0073	0.0083	0.8741	0.3821
IVOL(15.1754 <qi≤20.0446)	0.0223	0.0064	3.4605	0.0005
IVOL(20.0446<qi≤23.7586)	0.0092	0.0064	1.4518	0.1466
IVOL(qi>23.7586)	0.0309	0.0035	8.6997	0

Table 6: Threshold Effect Testing (2003-2007)

	Single threshold effect testing	**Double threshold effect testing**	**Triple threshold effect testing**
Threshold value	16.0593	12.6333	6.7252
F value	56.3595	12.7612	6.0721
P value	0.0000***	0.0000***	0.0033***
The critical value of F			
10%	2.537370552	2.34811693	2.678126032
5%	3.361838525	4.270672973	3.756017593
1%	7.567229521	8.220859076	5.781016819

Ps. *10% **代表 5% ***1% significant

Table 7: Threshold Parameters Estimation (2003-2007)

Regressor	**Coef**	**Std**	**t**	**prob**
Lnme	-0.1746	0.0365	-4.7806	0
Lnbeme	-0.7488	0.0475	-15.7518	0
Lnda	-0.4042	0.0594	-6.8017	0
Beta	0.0043	0.0004	10.0235	0
IVOL(qi≤ 6.7252)	-0.0328	0.0097	-3.3703	0.0008
IVOL(6.7252 <qi≤ 12.6333)	-0.018	0.0058	-3.1165	0.0018
IVOL(12.6333<qi≤16.0593)	-0.0015	0.0048	-0.3047	0.7606
IVOL(qi>16.0593)	0.0228	0.0022	10.3354	0

Table 8: Threshold Effect Testing (2008-2012)

	Single threshold effect testing	**Double threshold effect testing**	**Triple threshold effect testing**
Threshold value	14.5425	20.0419	8.2088
F value	19.9989	9.7663	1.5604
P value	0.0000***	0.0067***	0.2333
The critical value of F			
10%	2.743079087	2.800535769	2.564508557
5%	4.041343411	3.815625007	3.861734581
1%	7.315370585	8.65978215	7.074408676

Ps. *10% **代表 5% ***1% significant

Table 9: Threshold Parameters Estimation (2008-2012)

Regressor	**Coef**	**Std**	**t**	**prob**
Lnme	0.4134	0.0603	6.8579	0
Lnbeme	-0.505	0.0663	-7.6202	0
Lnda	-0.0247	0.0847	-0.2917	0.7705
Beta	0.0036	0.0005	7.8554	0
IVOL(qi≤14.5425)	0.0104	0.0063	1.6442	0.1002
IVOL(14.5425 <qi≤20.0419)	0.0341	0.0049	6.9756	0
IVOL(qi>20.0419)	0.0211	0.0029	7.2138	0

Appendix B: Likelihood Ratio Trend Graph

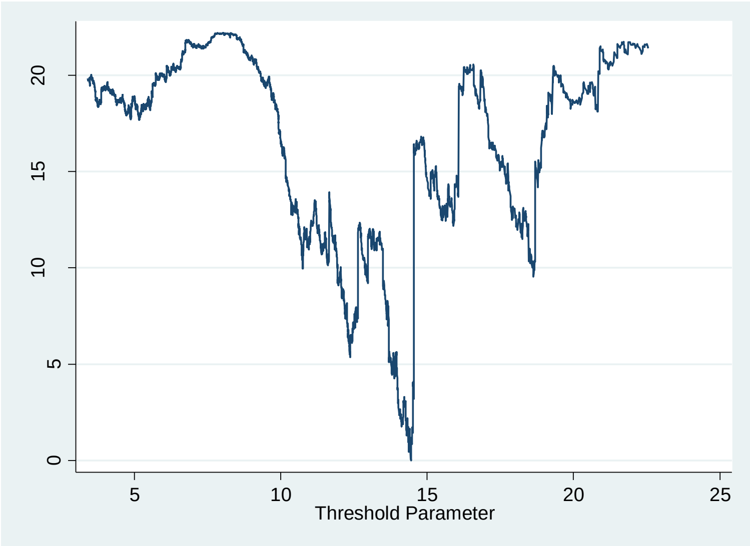

Figure 1: 1998-2012

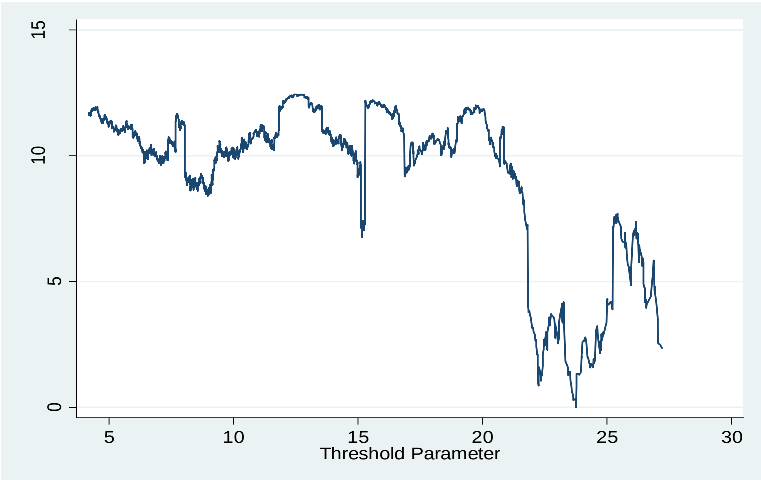

Figure 2: 1998-2002

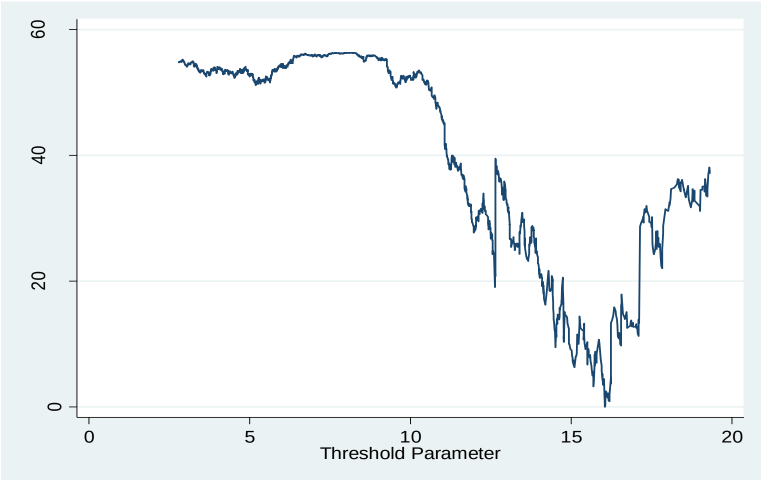

Figure 3: 2003-2007

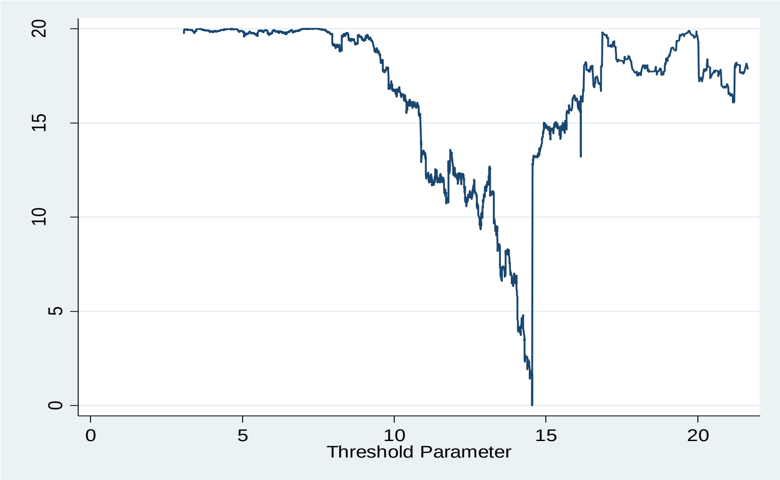

Figure 4: 2008-2012