

## **A Replenishment Model under a Multiple Supplier and Multiple Vehicle Environments**

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### ***Abstract***

*Production planning and inventory management has always been important for manufacturers. To be competitive in an ever-increasing fierce market, an outstanding replenishment control is essential for a firm to be responsive and cost competitive, so that decent profit can be acquired. The application of heuristics tools may be necessary to solve the ever-increasing complicated problem. This research considers the replenishment of multiple parts from multiple suppliers using multiple vehicles in multiple periods. The objectives are to minimize the total costs in the system and to determine an appropriate replenishment plan in each period. A mixed integer programming (MIP) model is presented first to solve the problem, and an artificial immune system (AIS) is constructed next to solve the problem when it becomes too complicated to solve. Then, a case study is used to examine the practicality of the proposed models. The results demonstrate that the proposed AIS model is an effective and accurate tool for devise a good replenishment plan for manufacturers.*

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**Key words:** *Replenishment, multiple traveling salesman problem (MTSP), mixed integer linear programming, artificial immune system (AIS).*

**JEL Classification:** *C 60, C 61, M11*

## 1. Introduction

Even though inventory management has been a popular topic in both the academic field and in real practice for a long time, the complexity of the production and supply chain environment has increased tremendously in the past few decades and has made the problem very difficult to solve. Conventional mathematical models, such as linear programming, nonlinear programming, mixed integer programming, may not be able to solve the NP-hard problem. Thus, heuristics tools may be used to solve such a complex problem.

Traveling salesman problem (TSP) has also been a popular research domain, and multiple traveling salesman problems (MTSP) are a generalization of the TSP. Through MTSP, a set of routes for several salesmen starting from and turning back to a depot can be determined (Bektas, 2006). For example, Kivelevitch *et al.* (2013) studied the multiple depots MTSP. The authors considered both the classical MTSP and the Min-Max MTSP. The former method is to minimize the sum of all tour lengths, and the latter is to minimize the longest tour. A task assignment algorithm was then proposed to obtain a market-based solution. Urrutia *et al.* (2015) studied a double traveling salesman problem with multiple stacks. In this problem, products are collected in the pickup route, put in one of the stacks in the vehicle, and then transported in the delivery route in the other network. The problem was solved by a dynamic programming based local search approach, and both the pickup route and the delivery route for a vehicle in the two different and disjoint networks could be determined.

In this research, the replenishment of multiple parts from multiple suppliers using multiple vehicles in multiple periods is studied, and the aim is to devise an appropriate replenishment plan in each period while minimizing the total costs in the system. Mixed integer programming (MIP) is applied first to solve the problem, and an artificial immune system (AIS) is proposed next to solve the problem. Both MIP and AIS are then used on a case study. The rest of this paper is organized as follows. In the next section, the MIP model and the AIS models are constructed. In section 3, a case study is presented. Some conclusions and discussions are made in the last section.

## 2. Proposed MIP Model

An MIP model is proposed to the multi-period, multi-supplier replenishment for multiple parts problem. Five major costs are considered: ordering cost, purchase cost, production cost, transportation cost, and holding cost. The total ordering cost in a planning horizon is as follows:

$$\text{Ordering Cost} = \sum_{i=1}^I \sum_{j=1}^J \sum_{t=1}^T o_{ij} \times Z_{ijt} \quad (1)$$

Where  $o_{ij}$  is the ordering cost of material  $j$  from supplier  $i$ ,  $Z_{ijt}$  shows if a purchase of material  $j$  from supplier  $i$  in period  $t$  is made.

The total purchase cost in a planning horizon is as follows:

$$\text{Purchase Cost} = \sum_{i=1}^I \sum_{j=1}^J \sum_{t=1}^T P(Q_{ijt}) \times Q_{ijt} \times Z_{ijt} \quad (2)$$

Where  $P(Q_{ijt})$  is the unit purchase cost of material  $j$  from supplier  $i$  in period  $t$ ,  $Q_{ijt}$  is the purchase quantity of material  $j$  from supplier  $i$  in period  $t$ , and  $Z_{ijt}$  shows if a purchase of material  $j$  from supplier  $i$  in period  $t$  is made.

The total production cost in a planning horizon is as follows:

$$\text{Production cost} = P = \sum_{g=1}^G \sum_{t=1}^T (F(S_{gt}) \times S_{gt} \times N_{gt}) \quad (3)$$

Where  $F(S_{gt})$  is the unit production cost of good  $g$  in period  $t$ ,  $S_{gt}$  shows if a production of good  $g$  is processed in period  $t$ , and  $N_{gt}$  represents whether a quantity is product of good  $g$  in period  $t$ .

The total transportation cost in a planning horizon is as follows:

$$\text{Transportation Cost} = \sum_{i=1}^I \sum_{i'=1, i' \neq i}^I \sum_{v=1}^V \sum_{t=1}^T \tau_{ii'} \times X_{ii'vt} \times \rho \quad (4)$$

where  $\tau_{ii'}$  is the transportation distance from node  $i$  to node  $i'$  each time,  $X_{ii'vt}$  shows if a shipment is made from node  $i$  to node  $i'$  with vehicle (route)  $v$  in period  $t$ ,  $\rho$  is the unit transportation cost per distance.

The total holding cost in a planning horizon is as follows:

$$\text{Holding Cost} = \sum_{t=1}^T \left[ \sum_{j=1}^J L_{jt} \times h_j + \sum_{g=1}^G F_{gt} \times h_g \right] \quad (5)$$

Where  $L_{jt}$  is the inventory level of material  $j$  in period  $t$  and  $h_j$  is the unit inventory holding cost of material  $j$  per period,  $F_{gt}$  is the inventory level of finished good  $g$  in period  $t$ , and  $h_g$  is the unit inventory holding cost finished good  $g$  per period.

In the MIP model, the objective is to minimize the total costs in a planning horizon, as follows:

*Minimize*

$$TC = \sum_{i=1}^I \sum_{j=1}^J \sum_{t=1}^T o_{ij} \times Z_{ijt} + \sum_{i=1}^I \sum_{j=1}^J \sum_{t=1}^T P(Q_{ijt}) \times Q_{ijt} \times Z_{ijt} + \sum_{g=1}^G \sum_{t=1}^T (F(S_{gt}) \times S_{gt} \times N_{gt}) + \sum_{i=0}^I \sum_{i'=0, i' \neq i}^I \sum_{v=1}^V \sum_{t=1}^T \lambda_{ii'} \times X_{ii'vt} \times \rho + \sum_{t=1}^T \left[ \sum_{j=1}^J L_{jt} \times h_j + \sum_{g=1}^G F_{gt} \times h_g \right] \quad (6)$$

### 3. Proposed AIS

Since its introduction, the AIS has been adopted in various fields, and it has been applied in the production and supply chain management studies to solve complicated problems. The steps of the AIS in this study are as follows (de Castro and Timmis, 2003; Huang et al., 2007; Naderi et al., 2010; Basu, 2012):

1. Analyze and decode the problem.
2. Generate an initial antibody population.
3. Calculate the affinity of antibodies.
4. Proliferate antibodies by cloning each member of the population depending on the affinity.
5. Mutate clone according to a predefined mutation rate.
6. Calculate the affinity of mutated clone.
7. Remove unfit antibodies in order to maintain the diversity of the population and to avoid the premature convergence.
8. Repeat steps 4 to 7 until a stopping rule is met.

## 4. Case Study

### 4.1 Data

Assume that a touch panel manufacturer manufactures two kinds of products and needs to purchase three kinds of material, touch panel (TP), glass and liquid crystal display (LCD). Each kind of material can be purchased from a single supplier, namely, S1, S2 and S3, respectively. There are three periods in a planning horizon, two vehicles in the system, and the transportation network is 3 by 3. The problem configuration is shown in Table 1. The inventory holding cost of the materials in a period are shown in Table 2. Table 3 shows the ordering cost of the materials from the respective supplier. Table 4 shows the bill of materials for the products. Table 5 shows the unit purchase cost of the materials from the respective supplier with different price breaks under all-units quantity discounts. Table 6 shows the transportation cost from one node to another node. Table 7 shows the unit production cost under different production types. Table 8 shows the quantity of each product demanded in each period.

**Table1: Problem configuration for the case study.**

Suppliers	Periods	Product types	Vehicle number	Transportation network
3	3	2	2	3 X 3

**Table2: Inventory holding cost ( $h_j$ )**

TP ( $j=1$ )	Glass ( $j=2$ )	LCD ( $j=3$ )
\$10	\$20	\$15

**Table3: Ordering cost ( $o_{ij}$ )**

$o_{ij}$	$j=1$	$j=2$	$j=3$
$i=1$	500		
$i=2$		350	
$i=3$			400

**Table4: Bill of materials ( $b_{gj}$ )**

$b_{gj}$	$j=1$	$j=2$	$j=3$
$g=1$	1	1	1
$g=2$	1		1

**Table5: Unit purchase cost under different price breaks ( $s_{ijk}$ )**

TP	Quantity	Price	Glass	Quantity	Price	LCD	Quantity	Price
S <sub>1</sub>	1-800	\$900	S <sub>2</sub>	1-1100	\$200	S <sub>3</sub>	1-500	\$500
	801-1200	\$800		1101-2200	\$170		501-1000	\$450
	1201-1400	\$700		2201-	\$150		1001-	\$400
	1401-	\$650						

**Table6: Transportation cost ( $r_{ii'}$ )**

	$i'=0$	$i'=1$	$i'=2$	$i'=3$
$i=0$	0	160	260	130
$i=1$	160	0	910	910
$i=2$	260	910	0	810
$i=3$	130	910	810	0

**Table7: Unit production cost under different production types ( $f_{it}$ ).**

Production type	Quantity range	Unit cost
In house	1-1000	\$100
Overtime	1001-1200	\$120
Outsourcing	1201-	\$170

**Table8: Product demand ( $d_t^g$ ) in different periods.**

Product (g) \ Period (t)	1	2	3
1	$d_1^1=151$	$d_2^1=302$	$d_3^1=501$
2	$d_2^2=101$		

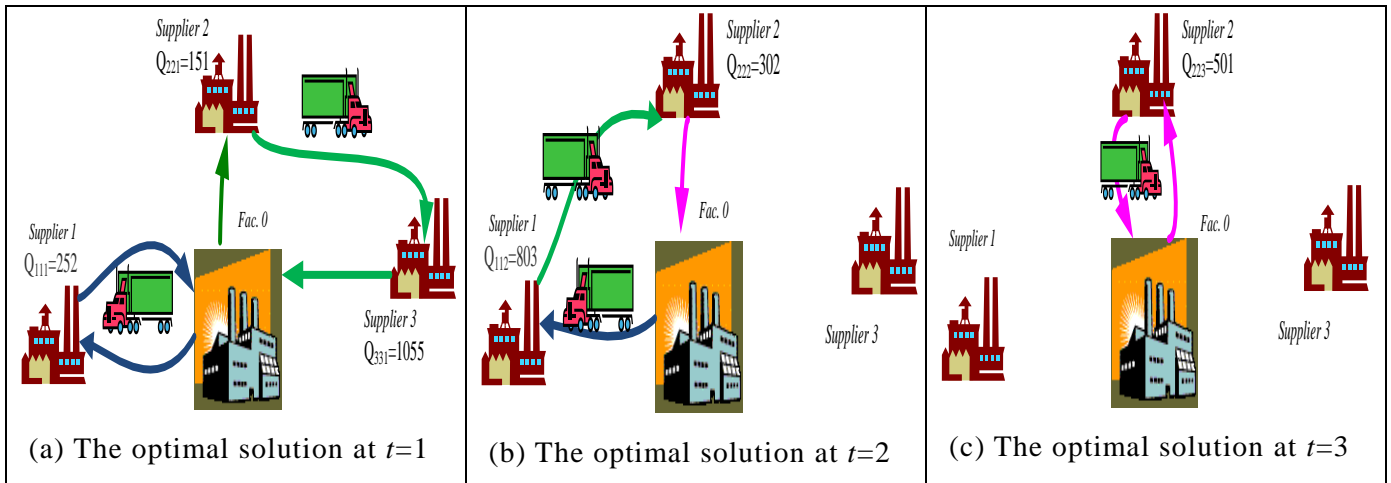
Both the MIP and the AIS are applied to solve the case. Under AIS, the number of iterations is set to be 100, and the affinity is set to be 0.6. The results of the beginning inventory, ending inventory and purchase quantity in each period from the MIP and the AIS are the same, as shown in Table 9. TP ( $j=1$ ) is purchased from supplier 1 in period 1 and 2, with 252 and 803 units, respectively. Glass ( $j=2$ ) is purchased from supplier 2 in period 1, 2 and 3, with 151, 302 and 501 units, respectively. LCD ( $j=3$ ) is purchased from supplier 3 in period, with 1055 units. The inventory level for each material in each period is shown by  $L_{jt}$ . For example, the inventory level for material LCD ( $j=3$ ) in period 1 is 803 units. The amount supplied for each kind of finished goods ( $g$ ) in each period ( $t$ ) is shown by  $S_t^g$ . For example, the amounts supplied for the first product ( $g=1$ ) in period 1, 2 and 3 are 151, 302, 501 units, respectively. The routing of a shipment by vehicle  $v$  in period  $t$  is shown by  $\pi_{vt}$ . For example,  $\pi_{11}:\{0,1,0\}$  indicates that vehicle 1 travels from the factory to supplier 1 and back to the factory in period 1, and  $\pi_{21}:\{0,2,3,0\}$  indicates that vehicle 2 travels from the factory to supplier 2, then to supplier 3, and back to the factory in period 1. The

routings under the MIP and the AI are the same, as shown in Figure 1. In addition, the relevant costs incurred under the MIP and the AI are the same, and the total cost is \$2,553,000. Among the total costs, total ordering Cost is \$2,450, total purchase cost is \$1,482,000, total production cost is \$949,500, total transportation cost is \$28,800, and total holding cost is \$ 90,250. The AI execution result is shown in Figure 2, and the convergence is reached in the 11<sup>th</sup> generations.

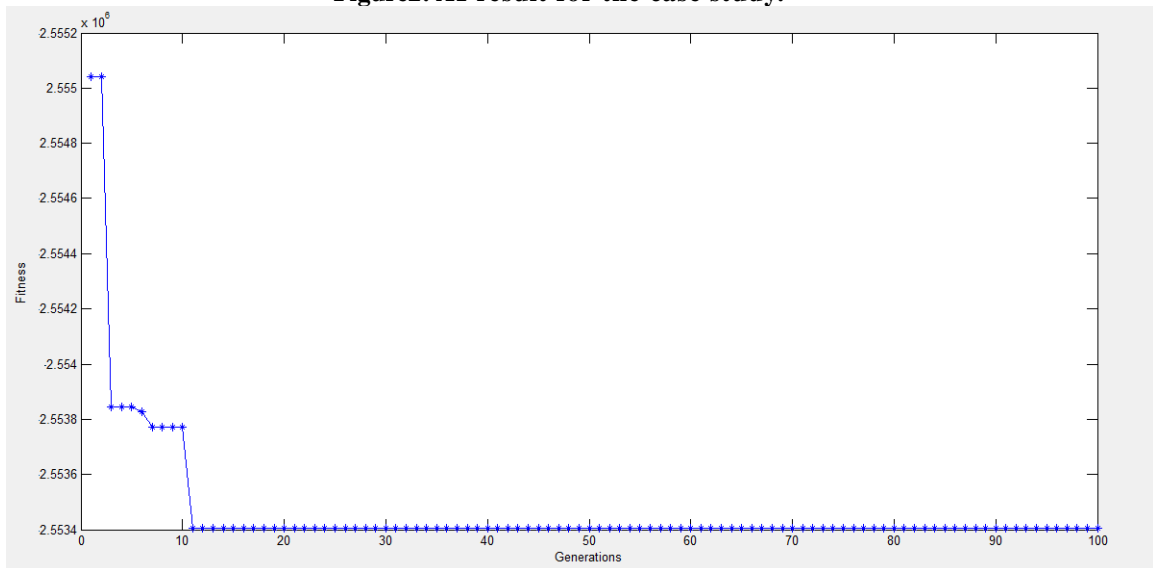
**Table 9: Relevant results in each period using the MIP and AIS**

Decision variables	t=1	t=2	t=3	Total cost
$Z_{ijt}$	$Z_{111}=1$ $Z_{221}=1$ $Z_{331}=1$	$Z_{112}=1$ $Z_{222}=1$	$Z_{223}=1$	
$Q_{ijt}$	$Q_{111}=252$ $Q_{221}=151$ $Q_{331}=1055$	$Q_{112}=803$ $Q_{222}=302$	$Q_{223}=501$	
$L_{jt}$	$L_{31} = 803$	$L_{12} = 501$ $L_{32} = 501$		\$2,553,000
$S_t^g$	$S_1^1 = 151$ $S_1^2 = 101$	$S_2^1 = 302$	$S_3^1 = 501$	
$\pi_{vt}$	$\pi_{11} : \{0,1,0\}$ $\pi_{21} : \{0,2,3,0\}$	$\pi_{12} : \{0,1,2,0\}$	$\pi_{13} : \{0,2,0\}$	

**Figure1: The optimal solution for the case study**



**Figure2: AI result for the case study.**



## 5. Conclusions and Recommendations

In this research, a replenishment problem which considers the production of multiple products using multiple parts from multiple suppliers using multiple vehicles in multiple periods is studied. Both the MIP and the AIS are adopted to solve the problem. In the case study, the optimal solution can be obtained using both models. In the future, more complicated cases will be studied. When the cases are too complicated to solve using the MIP, AIS shall help us obtain near-optimal solutions.

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