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Process Yield Index and Asymmetric Tolerances

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Abstract

Process yield has been an essential criterion used for manufacturing industries in business management. Quality managers and engineers involved in factories require capability analyses to access process yields, particularly, for processes with a very low fraction of defectives. It is noted that the accuracy of process yield is important; as it is one of major considerations for supplier selections in supply chains. In this paper, we investigate the capability index C_{pk} *which is a yield-based index and has gained considerable popularity as well as has been widely used in the manufacturing industry to provide numerical measures on process performance. However, the definition of index* C_{pk} *is independent of the target T of specification limits. That is, index* C_{pk} *cannot be used to access process yield for processes with asymmetric tolerances directly. To solve the problem, some investigations proposed modified process capability index based on* C_{pk} *for processes with asymmetric tolerances. In this paper, we focus on the index* C''_{pk} *in which the probability density function and cumulative distribution function of index* C''_{pk} *have been derived. We provide more extensive computations for lower confidence bound for processes with asymmetric tolerances. We also apply the calculation results to determine whether the process with asymmetric tolerance is capable or not for a real-world application.*

Key words: *Asymmetric tolerance, lower confidence bound, decision making.*
JEL Classification: *C 44, C46*

1. Introduction

Process capability analysis has become an important and integrated part in the applications of statistical process control to continuous improvement of quality and productivity ((Kane (1986), Chan *et al.* (1988), Pearn *et al.* (1992), Kotz and Lovelace (1998), Pearn and Chen (1998), and Kotz and Johnson (2002)). It quantifies the relationship between the actual process performance and the specification limits. It is noted that the capability index C_{pk} has been widely used and is a popular yield index. In current business companies, yield is an essential consideration and very close to profit of companies. Thus, the index C_{pk} considers the magnitude of process variance as well as the departures of process mean from the center. The index C_{pk} fails to distinguish between on-target and off-target processes for processes with asymmetric tolerances. Thus, Pearn and Chen (1998) proposed the index C''_{pk} for processes with asymmetric tolerance and it can be defined as:

$$C''_{pk} = \frac{d^* - \max\left\{d^*\left(\mu - T\right)/D_u, d^*\left(T-\mu\right)/D_l\right\}}{3\sigma}, \tag{1}$$

Where μ is the process mean, σ is the process standard deviation, T target value, $D_u = USL - T$, $D_l = T - LSL$, USL and LSL are the upper and lower specification limits, respectively, $d^* = \min\left\{D_u, D_l\right\}$, $A^* = \max\left\{d^*\left(\mu - T\right)/D_u,\ d^*\left(T-\mu\right)/D_l\right\}$. Obviously, for processes with symmetric tolerances ($D_u = D_l$), the C''_{pk} can be reduced to the C_{pk} index. Consequently, the index C''_{pk} is a generalization of C_{pk} index and it has been shown that is superior to other existing generalizations of C_{pk} index for processes with asymmetric tolerances.

Chang and Wu (2008) and Pearn and Wu (2012) have investigated the index C''_{pk} and provided the lower confidence bounds and non-conformities for processes with asymmetric tolerances. In this paper, we perform more extensive computations for the index C''_{pk} with regards to various asymmetric ratio, values of $\hat{C}''_{pk}$, and sample sizes. According to the computational results, the practitioners can determine whether the process is capable or not.

2. Process Capability Index C''_{pk} for Processes with Asymmetric Tolerance

Pearn and Chen (1998) proposed index C''_{pk}, a generalization of C_{pk} for processes with asymmetric tolerances. To estimate the generalization of C''_{pk} , Pearn and Chen (1998) considered the natural estimator $\hat{C}''_{pk}$ defined in the following. The natural estimator $\hat{C}''_{pk}$ is obtained by replacing the process mean μ and the process variance σ by their conventional estimators $\bar{X}$ and S^2, which may be obtained from a stable process.

$$\hat{C}''_{pk} = \frac{d^* - \hat{A}^*}{3S} , \tag{2}$$

Where $d^* = \min\{D_u, D_l\}$, $\hat{A}^* = \max\left\{d^*\left(\bar{X} - T\right)/D_u,\ d^*(T - \bar{X})/D_l\right\}$, $\bar{X} = \sum_{i=1}^{n} X_i / n$, and $S = \left[\sum_{i=1}^{n}(X_i - \bar{X})^2/(n-1)\right]^{1/2}$.

Pearn *et al*. (2004) also defined $B = n^{1/2}\left(d^*/\sigma\right)$, $K = \left(n-1\right)S^2/\sigma^2$, $Z = n^{1/2}\left(\bar{X} - T\right)/\sigma$, $Y = \left[\max\left\{\left(d^*/D_u\right)Z,\ -\left(d^*/D_l\right)Z\right\}\right]^2$, $\delta = n^{1/2}\left(\mu - T\right)/\sigma$. The estimator $\hat{C}''_{pk}$ can be written as:

$$\hat{C}''_{pk} = \frac{\sqrt{n-1}\left(B - \sqrt{Y}\right)}{3\sqrt{nK}} , \tag{3}$$

Pearn *et al*. (2004) presents that K is distributed as χ^2_{n-1}, a chi-square distribution with $n-1$ degrees of freedom, and Z is distributed as the normal distribution $N\left(\delta,1\right)$ with mean δ and variance 1 under the assumption of normality of X. Let $\Phi(\cdot)$ and $\phi(\cdot)$ be the cumulative distribution function and the probability density function of the standard normal distribution $N\left(0,1\right)$, respectively. The cumulative distribution function of Y can be expressed as:

$$F_Y\left(y\right) = \Phi\left[\left(D_u/d^*\right)\sqrt{y} - \delta\right] - \Phi\left[-\left(D_l/d^*\right)\sqrt{y} - \delta\right], \tag{4}$$

the probability density function of Y can be expressed as:

$$f_Y\left(y\right) = \frac{1}{2d^*\sqrt{y}}\left(D_u\phi\left[\left(D_u/d^*\right)\sqrt{y} - \delta\right] + D_l\phi\left[\left(D_l/d^*\right)\sqrt{y} + \delta\right]\right), \tag{5}$$

$F_Y\left(y\right)$ and $f_Y\left(y\right)$ can be used to derive the sampling distribution of C''_{pk}.

The cumulative distribution function of C''_{pk} can be obtained in the following:

$$F_{\hat{C}''_{pk}}(x) = \begin{cases} \int_{B^2}^{\infty} F_K\left(L(x,y)\right) f_Y\left(y\right) d_y, & x < 0, \\ 1 - F_Y\left(B^2\right), & x = 0, \\ 1 - \int_0^{B^2} F_K\left(L(x,y)\right) f_Y\left(y\right) d_y, & x > 0, \end{cases} \tag{6}$$

and the probability density function of C''_{pk} can be derived as:

$$f_{\hat{C}''_{pk}}(x)=\begin{cases}\int_{1}^{\infty} F_K\left(L\left(x,B^2t\right)\right) f_Y\left(B^2t\right)\frac{2L\left(x,B^2t\right)}{-x}dt, & x<0,\\ \int_{0}^{1} F_K\left(L\left(x,B^2t\right)\right) f_Y\left(B^2t\right)\frac{2L\left(x,B^2t\right)}{x}dt, & x>0,\end{cases} \tag{7}$$

where $B=n^{1/2}\left(d^*/\sigma\right)$, $L(x,y)=(n-1)\left(B-y^{1/2}\right)^2/\left(9nx^2\right)$, $F_K(\cdot)$ is the cumulative distribution function of K, $f_K(\cdot)$ is the probability density function of K, $F_Y(\cdot)$ is the cumulative distribution function of Y expressed as Eq. (6), and $f_Y(\cdot)$ is the probability density function of Y expressed as Eq. (7).

3. Lower confidence bounds on C''_{pk}

The development of the lower confidence bound on the actual manufacturing capability is essential. The lower confidence bound not only gives us a clue about the minimal level of the actual manufacturing performance, which is closely related to the fraction of nonconforming units (defectives), but is also useful in making decisions for manufacturing capability testing (Chang and Wu, 2008). For process with a target value not set to the mid-point of the manufacturing specification ($\mathrm{T}\neq m$), we consider four asymmetric cases.

3.1 Lower confidence bounds C and parameter ξ

In this paper, we develop MatLab program to compute the lower confidence bound in four cases, and we present the results in Table 1(a)-1(d) to show the lower confidence bounds C versus ξ =0.0(-0.1)-1.0 or 0.0(0.1)1.0, and sample size *n*=10(10)100 for $\hat{C}''_{pk}$ =0.70 for $T\geq m$ or $T\leq m$ and α-risk=0.05.

To eliminate the need to further estimate the parameter ξ, we examine the behavior of the lower confidence bound C as the function of the process characteristic ξ. We perform extensive computations to calculate the lower confidence bound C versus -1.00≤ξ≤1.00, and *n*=20(20)200 for $\hat{C}''_{pk}$ =0.70, α-risk=0.05 and ratio=1/9, 2/8, 3/7, 4/6, 5/5 or 5/5, 6/4, 7/3, 8/2, 9/1. It is noted that the behavior of the lower confidence bounds C against the parameter ξ, we find out that the lower confidence bounds C (i) is decreasing in $\xi\neq 0$, and is increasing in *n*, (ii) obtains its minimal at ξ =1 in $\mathrm{T}\geq\mathrm{m}$ cases, or ξ = -1 in $\mathrm{T}\leq\mathrm{m}$ cases, and (iii) stays the same for $\xi=\pm 1$ for all C (with accuracy up to 10^{-6}). Further, we know that (iv) two groups of the lower confidence bound C are the same. One is the lower confidence bound C with ξ =1 for $T\geq m$ and ξ = -1 for $T\leq m$, and the other one is the lower confidence bound C with ξ = -1 for $T\geq m$ and ξ = 1 for $T\leq m$. Hence, for practical purpose we can obtain the lower confidence bound with ξ =1 for $T \geq m$ or ξ = -1 for $T \leq m$, without having to further estimate the parameter ξ. This approach ensures that the actual manufacturing capability based on those lower confidence bounds is indeed more reliable than all existing methods.

In the paper, we obtain the similar computation results with Chang and Wu (2008). And we also find that the lower confidence bound C is more reliable with ξ =1 for $T \geq m$ and ξ = -1 for $T \leq m$ or with ξ = -1 for $T \geq m$ and ξ = 1 for $T \leq m$. However, in the further study, it is worth to discuss whether $\xi = \pm 1$ are the most appropriate settings.

4. Applications

In the paper, we consider a real-world case to demonstrate the applicability of the proposed method. The example has been presented in Lin and Pearn (2002), in which the company produces the high-end audio speaker components including rubber edge, Pulux edge, Kevlar cone, honeycomb and many others. It is noted that the lower specification limit, target, and upper specification limit are 5.650, 5.835, and 5.950, respectively. The process is asymmetric tolerance since the mid-point $m = (USL + LSL)/2 = 5.800$, and $T = 5.835$. In this case, we have $D_u = USL - T = 0.115$, $D_l = T - LSL = 0.185$, and $d^* = \min\{D_u, D_l\} = 0.115$. Ninety observations were collected. We set the preset process requirement is that $C = 1.33$. We can calculate $\bar{X} = \sum_{i=1}^{n} X_i / n =$ 5.83033, $S = \left[\sum_{i=1}^{n} (X_i - \bar{X})^2 / (n-1)\right]^{1/2}$ =0.02334, $\hat{A}^* = \max\{d^*(\bar{X} - T)/D_u,\ d^*(T - \bar{X})/D_l\} =$ 0.00290, $\hat{\xi} = (\bar{X} - T)/S =$ -0.20009 and $\hat{C}''_{pk} = (d^* - \hat{A}^*)/3S = 1.60$.

In the case, based on the confidence level γ =0.95, $\hat{C}''_{pk} = 1.60$, unknown parameter $\xi = 1$, n =90, the lower confidence bound of C''_{pk} can be computed or can be seen in Table 2. The lower confidence bund LCB =1.393 is obtained. Therefore, we conclude that the true value of the process capability index C''_{pk} is no less than 1.393 with 95% level of confidence. The Pulux edge manufacturing process is capable and the corresponding maximum non-conformity is no greater than 33 PPM.

Table 1(a). Lower confidence bounds C for $\hat{C}''_{pk}$ =0.70 with ξ =0.0(-0.1)-1.0, and sample n=10(10)100 for $\mathrm{T} \geq \mathrm{m}$ and α-risk=0.05

Co=0.7 alpha=0.05 r=9/1

ξ	n=10	20	30	40	50	60	70	80	90	100
0	0.457	0.534	0.566	0.584	0.597	0.606	0.613	0.619	0.624	0.628
-0.1	0.447	0.523	0.556	0.575	0.588	0.597	0.605	0.611	0.616	0.620
-0.2	0.439	0.517	0.550	0.570	0.584	0.594	0.602	0.608	0.613	0.618
-0.3	0.433	0.513	0.548	0.568	0.582	0.593	0.601	0.607	0.613	0.617
-0.4	0.429	0.511	0.547	0.568	0.582	0.593	0.601	0.607	0.613	0.617
-0.5	0.427	0.511	0.547	0.568	0.582	0.593	0.601	0.607	0.613	0.617
-0.6	0.426	0.510	0.547	0.568	0.582	0.593	0.601	0.607	0.613	0.617
-0.7	0.425	0.510	0.547	0.568	0.582	0.593	0.601	0.607	0.613	0.617
-0.8	0.425	0.510	0.547	0.568	0.582	0.593	0.601	0.607	0.613	0.617
-0.9	0.425	0.510	0.547	0.568	0.582	0.593	0.601	0.607	0.613	0.617
-1	0.425	0.510	0.547	0.568	0.582	0.593	0.601	0.607	0.613	0.617

Co=0.7 alpha=0.05 r=8/2

ξ	n=10	20	30	40	50	60	70	80	90	100
0	0.464	0.539	0.570	0.588	0.600	0.609	0.616	0.622	0.626	0.630
-0.1	0.451	0.526	0.557	0.576	0.589	0.598	0.605	0.611	0.616	0.620
-0.2	0.441	0.517	0.550	0.570	0.583	0.593	0.601	0.607	0.612	0.617
-0.3	0.434	0.512	0.547	0.567	0.581	0.592	0.600	0.606	0.612	0.616
-0.4	0.429	0.510	0.545	0.567	0.581	0.591	0.600	0.606	0.612	0.616
-0.5	0.425	0.509	0.545	0.566	0.581	0.591	0.600	0.606	0.612	0.616
-0.6	0.424	0.508	0.545	0.566	0.581	0.591	0.600	0.606	0.612	0.616
-0.7	0.423	0.508	0.545	0.566	0.581	0.591	0.600	0.606	0.612	0.616
-0.8	0.422	0.508	0.545	0.566	0.581	0.591	0.600	0.606	0.612	0.616
-0.9	0.422	0.508	0.545	0.566	0.581	0.591	0.600	0.606	0.612	0.616
-1	0.422	0.508	0.545	0.566	0.581	0.591	0.600	0.606	0.612	0.616

Co=0.7 alpha=0.05 r=7/3

ξ	n=10	20	30	40	50	60	70	80	90	100
0	0.472	0.544	0.574	0.592	0.604	0.612	0.619	0.624	0.629	0.632
-0.1	0.455	0.528	0.558	0.577	0.589	0.598	0.605	0.611	0.615	0.619
-0.2	0.442	0.517	0.549	0.568	0.581	0.591	0.599	0.605	0.610	0.615
-0.3	0.432	0.510	0.544	0.564	0.578	0.589	0.597	0.604	0.609	0.614
-0.4	0.425	0.506	0.542	0.563	0.578	0.589	0.597	0.604	0.609	0.614
-0.5	0.420	0.504	0.541	0.563	0.578	0.589	0.597	0.604	0.609	0.614
-0.6	0.418	0.503	0.541	0.563	0.578	0.589	0.597	0.604	0.609	0.614
-0.7	0.416	0.503	0.541	0.563	0.578	0.589	0.597	0.604	0.609	0.614
-0.8	0.415	0.503	0.541	0.563	0.578	0.589	0.597	0.604	0.609	0.614
-0.9	0.415	0.503	0.541	0.563	0.578	0.589	0.597	0.604	0.609	0.614
-1	0.415	0.503	0.541	0.563	0.578	0.589	0.597	0.604	0.609	0.614

Co=0.7 alpha=0.05 r=6/4

ξ	n=10	20	30	40	50	60	70	80	90	100
0	0.481	0.550	0.579	0.596	0.608	0.616	0.622	0.627	0.632	0.635
-0.1	0.459	0.529	0.558	0.576	0.588	0.597	0.603	0.609	0.613	0.617
-0.2	0.441	0.513	0.545	0.564	0.577	0.587	0.595	0.601	0.606	0.611
-0.3	0.426	0.503	0.537	0.558	0.572	0.583	0.592	0.599	0.605	0.610
-0.4	0.416	0.497	0.534	0.556	0.571	0.583	0.592	0.599	0.605	0.610
-0.5	0.409	0.494	0.533	0.556	0.571	0.583	0.592	0.599	0.605	0.610
-0.6	0.405	0.493	0.532	0.555	0.571	0.583	0.592	0.599	0.605	0.610
-0.7	0.402	0.493	0.532	0.555	0.571	0.583	0.592	0.599	0.605	0.610
-0.8	0.401	0.493	0.532	0.555	0.571	0.583	0.592	0.599	0.605	0.610
-0.9	0.400	0.493	0.532	0.555	0.571	0.583	0.592	0.599	0.605	0.610
-1	0.400	0.493	0.532	0.555	0.571	0.583	0.592	0.599	0.605	0.610

Co=0.7 alpha=0.05 r=5/5

ξ	n=10	20	30	40	50	60	70	80	90	100
0	0.491	0.557	0.585	0.601	0.612	0.620	0.626	0.631	0.635	0.638
-0.1	0.460	0.527	0.556	0.573	0.584	0.593	0.599	0.605	0.609	0.613
-0.2	0.434	0.505	0.536	0.555	0.568	0.578	0.586	0.592	0.598	0.603
-0.3	0.413	0.489	0.524	0.545	0.561	0.572	0.581	0.589	0.595	0.601
-0.4	0.397	0.480	0.518	0.542	0.559	0.571	0.581	0.589	0.595	0.601
-0.5	0.386	0.475	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.6	0.379	0.473	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.7	0.374	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.8	0.372	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.9	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-1	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601

Table 1(b). Lower confidence bounds C for $\hat{C}''_{pk}$ =0.70 with ξ =0.0(-0.1)-1.0, and sample n = 10(10)100 for $T \le m$ and α-risk=0.05

Co=0.7 alpha=0.05 r=5/5

ξ	n=10	20	30	40	50	60	70	80	90	100
0	0.491	0.557	0.585	0.601	0.612	0.620	0.626	0.631	0.635	0.638
-0.1	0.460	0.527	0.556	0.573	0.584	0.593	0.599	0.605	0.609	0.613
-0.2	0.434	0.505	0.536	0.555	0.568	0.578	0.586	0.592	0.598	0.603
-0.3	0.413	0.489	0.524	0.545	0.561	0.572	0.581	0.589	0.595	0.601
-0.4	0.397	0.480	0.518	0.542	0.559	0.571	0.581	0.589	0.595	0.601
-0.5	0.386	0.475	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.6	0.379	0.473	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.7	0.374	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.8	0.372	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.9	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-1	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601

Co=0.7 alpha=0.05 r=4/6

ξ	n=10	20	30	40	50	60	70	80	90	100
0	0.481	0.550	0.579	0.596	0.608	0.616	0.622	0.627	0.632	0.635
-0.1	0.453	0.523	0.553	0.570	0.582	0.591	0.598	0.603	0.608	0.612
-0.2	0.429	0.502	0.534	0.553	0.567	0.577	0.585	0.592	0.597	0.602
-0.3	0.409	0.487	0.523	0.545	0.560	0.572	0.581	0.589	0.595	0.601
-0.4	0.395	0.479	0.518	0.542	0.559	0.571	0.581	0.589	0.595	0.601
-0.5	0.384	0.474	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.6	0.378	0.473	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.7	0.374	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.8	0.372	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.9	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-1	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601

Co=0.7 alpha=0.05 r=3/7

ξ	n=10	20	30	40	50	60	70	80	90	100
0	0.472	0.544	0.574	0.592	0.604	0.612	0.619	0.624	0.629	0.632
-0.1	0.446	0.519	0.549	0.568	0.580	0.589	0.596	0.602	0.607	0.611
-0.2	0.424	0.499	0.532	0.552	0.566	0.576	0.585	0.591	0.597	0.602
-0.3	0.406	0.486	0.522	0.544	0.560	0.572	0.581	0.589	0.595	0.601
-0.4	0.392	0.478	0.517	0.542	0.559	0.571	0.581	0.589	0.595	0.601
-0.5	0.383	0.474	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.6	0.377	0.473	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.7	0.373	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.8	0.372	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.9	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-1	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601

Co=0.7 alpha=0.05 r=2/8

ξ	n=10	20	30	40	50	60	70	80	90	100
0	0.464	0.539	0.570	0.588	0.600	0.609	0.616	0.622	0.626	0.630
-0.1	0.440	0.515	0.547	0.565	0.578	0.588	0.595	0.601	0.606	0.610
-0.2	0.419	0.497	0.531	0.551	0.565	0.576	0.584	0.591	0.597	0.602
-0.3	0.403	0.485	0.521	0.544	0.560	0.572	0.581	0.589	0.595	0.601
-0.4	0.390	0.477	0.517	0.542	0.559	0.571	0.581	0.589	0.595	0.601
-0.5	0.382	0.474	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.6	0.376	0.473	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.7	0.373	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.8	0.372	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.9	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-1	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601

Co=0.7 alpha=0.05 r=1/9

ξ	n=10	20	30	40	50	60	70	80	90	100
0	0.457	0.534	0.566	0.584	0.597	0.606	0.613	0.619	0.624	0.628
-0.1	0.435	0.512	0.544	0.564	0.577	0.586	0.594	0.600	0.605	0.609
-0.2	0.415	0.495	0.529	0.550	0.565	0.575	0.584	0.591	0.597	0.602
-0.3	0.400	0.483	0.521	0.544	0.560	0.572	0.581	0.589	0.595	0.601
-0.4	0.388	0.477	0.517	0.542	0.558	0.571	0.581	0.589	0.595	0.601
-0.5	0.381	0.474	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.6	0.376	0.473	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.7	0.373	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.8	0.372	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-0.9	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
-1	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601

Table 1(c). Lower confidence bounds C for $\hat{C}''_{pk}$ =0.70 with ξ =0.0(0.1)1.0, and sample n = 10(10)100 for $T \geq m$ and α-risk=0.05

Co=0.7 alpha=0.05 r=9/1

ξ	n=10	20	30	40	50	60	70	80	90	100
0	0.457	0.534	0.566	0.584	0.597	0.606	0.613	0.619	0.624	0.628
0.1	0.435	0.512	0.544	0.564	0.577	0.586	0.594	0.600	0.605	0.609
0.2	0.415	0.495	0.529	0.550	0.565	0.575	0.584	0.591	0.597	0.602
0.3	0.400	0.483	0.521	0.544	0.560	0.572	0.581	0.589	0.595	0.601
0.4	0.388	0.477	0.517	0.542	0.558	0.571	0.581	0.589	0.595	0.601
0.5	0.381	0.474	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.6	0.376	0.473	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.7	0.373	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.8	0.372	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.9	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
1	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601

Co=0.7 alpha=0.05 r=8/2

ξ	n=10	20	30	40	50	60	70	80	90	100
0	0.464	0.539	0.570	0.588	0.600	0.609	0.616	0.622	0.626	0.630
0.1	0.440	0.515	0.547	0.565	0.578	0.588	0.595	0.601	0.606	0.610
0.2	0.419	0.497	0.531	0.551	0.565	0.576	0.584	0.591	0.597	0.602
0.3	0.403	0.485	0.521	0.544	0.560	0.572	0.581	0.589	0.595	0.601
0.4	0.390	0.477	0.517	0.542	0.559	0.571	0.581	0.589	0.595	0.601
0.5	0.382	0.474	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.6	0.376	0.473	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.7	0.373	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.8	0.372	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.9	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
1	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601

Co=0.7 alpha=0.05 r=7/3

ξ	n=10	20	30	40	50	60	70	80	90	100
0	0.472	0.544	0.574	0.592	0.604	0.612	0.619	0.624	0.629	0.632
0.1	0.446	0.519	0.549	0.568	0.580	0.589	0.596	0.602	0.607	0.611
0.2	0.424	0.499	0.532	0.552	0.566	0.576	0.585	0.591	0.597	0.602
0.3	0.406	0.486	0.522	0.544	0.560	0.572	0.581	0.589	0.595	0.601
0.4	0.392	0.478	0.517	0.542	0.559	0.571	0.581	0.589	0.595	0.601
0.5	0.383	0.474	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.6	0.377	0.473	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.7	0.373	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.8	0.372	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.9	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
1	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601

Co=0.7 alpha=0.05 r=6/4

ξ	n=10	20	30	40	50	60	70	80	90	100
0	0.481	0.550	0.579	0.596	0.608	0.616	0.622	0.627	0.632	0.635
0.1	0.453	0.523	0.553	0.570	0.582	0.591	0.598	0.603	0.608	0.612
0.2	0.429	0.502	0.534	0.553	0.567	0.577	0.585	0.592	0.597	0.602
0.3	0.409	0.487	0.523	0.545	0.560	0.572	0.581	0.589	0.595	0.601
0.4	0.395	0.479	0.518	0.542	0.559	0.571	0.581	0.589	0.595	0.601
0.5	0.384	0.474	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.6	0.378	0.473	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.7	0.374	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.8	0.372	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.9	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
1	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601

Co=0.7 alpha=0.05 r=5/5

ξ	n=10	20	30	40	50	60	70	80	90	100
0	0.491	0.557	0.585	0.601	0.612	0.620	0.626	0.631	0.635	0.638
0.1	0.460	0.527	0.556	0.573	0.584	0.593	0.599	0.605	0.609	0.613
0.2	0.434	0.505	0.536	0.555	0.568	0.578	0.586	0.592	0.598	0.603
0.3	0.413	0.489	0.524	0.545	0.561	0.572	0.581	0.589	0.595	0.601
0.4	0.397	0.480	0.518	0.542	0.559	0.571	0.581	0.589	0.595	0.601
0.5	0.386	0.475	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.6	0.379	0.473	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.7	0.374	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.8	0.372	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.9	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
1	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601

Table 1(d). Lower confidence bounds C for $\hat{C}''_{pk}$ =0.70 with ξ =0.0(0.1)1.0, and sample n = 10(10)100 for $T \leq m$ and α-risk=0.05

Co=0.7 alpha=0.05 r=5/5

ξ	n=10	20	30	40	50	60	70	80	90	100
0	0.491	0.557	0.585	0.601	0.612	0.620	0.626	0.631	0.635	0.638
0.1	0.460	0.527	0.556	0.573	0.584	0.593	0.599	0.605	0.609	0.613
0.2	0.434	0.505	0.536	0.555	0.568	0.578	0.586	0.592	0.598	0.603
0.3	0.413	0.489	0.524	0.545	0.561	0.572	0.581	0.589	0.595	0.601
0.4	0.397	0.480	0.518	0.542	0.559	0.571	0.581	0.589	0.595	0.601
0.5	0.386	0.475	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.6	0.379	0.473	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.7	0.374	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.8	0.372	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
0.9	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601
1	0.371	0.472	0.516	0.541	0.558	0.571	0.581	0.589	0.595	0.601

Co=0.7 alpha=0.05 r=4/6

ξ	n=10	20	30	40	50	60	70	80	90	100
0	0.481	0.550	0.579	0.596	0.608	0.616	0.622	0.627	0.632	0.635
0.1	0.459	0.529	0.558	0.576	0.588	0.597	0.603	0.609	0.613	0.617
0.2	0.441	0.513	0.545	0.564	0.577	0.587	0.595	0.601	0.606	0.611
0.3	0.426	0.503	0.537	0.558	0.572	0.583	0.592	0.599	0.605	0.610
0.4	0.416	0.497	0.534	0.556	0.571	0.583	0.592	0.599	0.605	0.610
0.5	0.409	0.494	0.533	0.556	0.571	0.583	0.592	0.599	0.605	0.610
0.6	0.405	0.493	0.532	0.555	0.571	0.583	0.592	0.599	0.605	0.610
0.7	0.402	0.493	0.532	0.555	0.571	0.583	0.592	0.599	0.605	0.610
0.8	0.401	0.493	0.532	0.555	0.571	0.583	0.592	0.599	0.605	0.610
0.9	0.400	0.493	0.532	0.555	0.571	0.583	0.592	0.599	0.605	0.610
1	0.400	0.493	0.532	0.555	0.571	0.583	0.592	0.599	0.605	0.610

Co=0.7 alpha=0.05 r=3/7

ξ	n=10	20	30	40	50	60	70	80	90	100
0	0.472	0.544	0.574	0.592	0.604	0.612	0.619	0.624	0.629	0.632
0.1	0.455	0.528	0.558	0.577	0.589	0.598	0.605	0.611	0.615	0.619
0.2	0.442	0.517	0.549	0.568	0.581	0.591	0.599	0.605	0.610	0.615
0.3	0.432	0.510	0.544	0.564	0.578	0.589	0.597	0.604	0.609	0.614
0.4	0.425	0.506	0.542	0.563	0.578	0.589	0.597	0.604	0.609	0.614
0.5	0.420	0.504	0.541	0.563	0.578	0.589	0.597	0.604	0.609	0.614
0.6	0.418	0.503	0.541	0.563	0.578	0.589	0.597	0.604	0.609	0.614
0.7	0.416	0.503	0.541	0.563	0.578	0.589	0.597	0.604	0.609	0.614
0.8	0.415	0.503	0.541	0.563	0.578	0.589	0.597	0.604	0.609	0.614
0.9	0.415	0.503	0.541	0.563	0.578	0.589	0.597	0.604	0.609	0.614
1	0.415	0.503	0.541	0.563	0.578	0.589	0.597	0.604	0.609	0.614

Co=0.7 alpha=0.05 r=2/8

ξ	n=10	20	30	40	50	60	70	80	90	100
0	0.464	0.539	0.570	0.588	0.600	0.609	0.616	0.622	0.626	0.630
0.1	0.451	0.526	0.557	0.576	0.589	0.598	0.605	0.611	0.616	0.620
0.2	0.441	0.517	0.550	0.570	0.583	0.593	0.601	0.607	0.612	0.617
0.3	0.434	0.512	0.547	0.567	0.581	0.592	0.600	0.606	0.612	0.616
0.4	0.429	0.510	0.545	0.567	0.581	0.591	0.600	0.606	0.612	0.616
0.5	0.425	0.509	0.545	0.566	0.581	0.591	0.600	0.606	0.612	0.616
0.6	0.424	0.508	0.545	0.566	0.581	0.591	0.600	0.606	0.612	0.616
0.7	0.423	0.508	0.545	0.566	0.581	0.591	0.600	0.606	0.612	0.616
0.8	0.422	0.508	0.545	0.566	0.581	0.591	0.600	0.606	0.612	0.616
0.9	0.422	0.508	0.545	0.566	0.581	0.591	0.600	0.606	0.612	0.616
1	0.422	0.508	0.545	0.566	0.581	0.591	0.600	0.606	0.612	0.616

Co=0.7 alpha=0.05 r=1/9

ξ	n=10	20	30	40	50	60	70	80	90	100
0	0.457	0.534	0.566	0.584	0.597	0.606	0.613	0.619	0.624	0.628
0.1	0.447	0.523	0.556	0.575	0.588	0.597	0.605	0.611	0.616	0.620
0.2	0.439	0.517	0.550	0.570	0.584	0.594	0.602	0.608	0.613	0.618
0.3	0.433	0.513	0.548	0.568	0.582	0.593	0.601	0.607	0.613	0.617
0.4	0.429	0.511	0.547	0.568	0.582	0.593	0.601	0.607	0.613	0.617
0.5	0.427	0.511	0.547	0.568	0.582	0.593	0.601	0.607	0.613	0.617
0.6	0.426	0.510	0.547	0.568	0.582	0.593	0.601	0.607	0.613	0.617
0.7	0.425	0.510	0.547	0.568	0.582	0.593	0.601	0.607	0.613	0.617
0.8	0.425	0.510	0.547	0.568	0.582	0.593	0.601	0.607	0.613	0.617
0.9	0.425	0.510	0.547	0.568	0.582	0.593	0.601	0.607	0.613	0.617
1	0.425	0.510	0.547	0.568	0.582	0.593	0.601	0.607	0.613	0.617

Table2 Lower confidence bounds γ of C''_{pk} for $\hat{C}''_{pk}$ =0.70(0.10)1.8, n=10(5)200 and confidence level γ =0.95.

n	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
10	0.371	0.437	0.503	0.568	0.632	0.695	0.759	0.821	0.884	0.947	1.009	1.071
15	0.435	0.508	0.581	0.652	0.723	0.794	0.865	0.935	1.005	1.075	1.145	1.214
20	0.472	0.549	0.626	0.701	0.777	0.852	0.926	1.001	1.075	1.149	1.223	1.297
25	0.497	0.577	0.656	0.734	0.812	0.890	0.968	1.045	1.122	1.199	1.276	1.353
30	0.516	0.597	0.678	0.758	0.838	0.918	0.998	1.077	1.156	1.235	1.314	1.393
35	0.530	0.613	0.695	0.777	0.859	0.940	1.021	1.102	1.183	1.263	1.344	1.425
40	0.541	0.625	0.709	0.792	0.875	0.957	1.040	1.122	1.204	1.286	1.368	1.450
45	0.550	0.635	0.720	0.804	0.888	0.971	1.055	1.138	1.221	1.304	1.387	1.470
50	0.558	0.644	0.729	0.814	0.899	0.984	1.068	1.152	1.236	1.320	1.404	1.488
55	0.565	0.652	0.738	0.823	0.909	0.994	1.079	1.164	1.249	1.333	1.418	1.503
60	0.571	0.658	0.745	0.831	0.917	1.003	1.089	1.174	1.260	1.345	1.430	1.515
65	0.576	0.664	0.751	0.838	0.924	1.011	1.097	1.183	1.269	1.355	1.441	1.527
70	0.581	0.669	0.756	0.844	0.931	1.018	1.105	1.191	1.278	1.364	1.451	1.537
75	0.585	0.673	0.761	0.849	0.937	1.024	1.111	1.198	1.285	1.372	1.459	1.546
80	0.589	0.677	0.766	0.854	0.942	1.030	1.117	1.205	1.292	1.380	1.467	1.554
85	0.592	0.681	0.770	0.858	0.947	1.035	1.123	1.211	1.299	1.386	1.474	1.562
90	0.595	0.685	0.774	0.863	0.951	1.040	1.128	1.216	1.304	1.393	1.481	1.569
95	0.598	0.688	0.777	0.866	0.955	1.044	1.133	1.221	1.310	1.398	1.487	1.575
100	0.601	0.691	0.780	0.870	0.959	1.048	1.137	1.226	1.315	1.403	1.492	1.581
105	0.603	0.693	0.783	0.873	0.962	1.052	1.141	1.230	1.319	1.408	1.497	1.586
110	0.605	0.696	0.786	0.876	0.966	1.055	1.145	1.234	1.323	1.413	1.502	1.591
115	0.607	0.698	0.788	0.879	0.969	1.058	1.148	1.238	1.327	1.417	1.506	1.596
120	0.609	0.700	0.791	0.881	0.971	1.061	1.151	1.241	1.331	1.421	1.510	1.600
125	0.611	0.702	0.793	0.884	0.974	1.064	1.154	1.245	1.335	1.424	1.514	1.604
130	0.613	0.704	0.795	0.886	0.977	1.067	1.157	1.248	1.338	1.428	1.518	1.608
135	0.615	0.706	0.797	0.888	0.979	1.070	1.160	1.250	1.341	1.431	1.521	1.612
140	0.616	0.708	0.799	0.890	0.981	1.072	1.163	1.253	1.344	1.434	1.525	1.615
145	0.618	0.709	0.801	0.892	0.983	1.074	1.165	1.256	1.347	1.437	1.528	1.618
150	0.619	0.711	0.802	0.894	0.985	1.076	1.167	1.258	1.349	1.440	1.531	1.621
155	0.620	0.712	0.804	0.896	0.987	1.078	1.169	1.261	1.352	1.443	1.534	1.624
160	0.622	0.714	0.806	0.897	0.989	1.080	1.172	1.263	1.354	1.445	1.536	1.627
165	0.623	0.715	0.807	0.899	0.991	1.082	1.174	1.265	1.356	1.448	1.539	1.630
170	0.624	0.716	0.808	0.900	0.992	1.084	1.175	1.267	1.358	1.450	1.541	1.632
175	0.625	0.718	0.810	0.902	0.994	1.086	1.177	1.269	1.360	1.452	1.543	1.635
180	0.626	0.719	0.811	0.903	0.995	1.087	1.179	1.271	1.362	1.454	1.546	1.637
185	0.627	0.720	0.812	0.905	0.997	1.089	1.181	1.273	1.364	1.456	1.548	1.639
190	0.628	0.721	0.813	0.906	0.998	1.090	1.182	1.274	1.366	1.458	1.550	1.642
195	0.629	0.722	0.815	0.907	0.999	1.092	1.184	1.276	1.368	1.460	1.552	1.644
200	0.630	0.723	0.816	0.908	1.001	1.093	1.185	1.277	1.370	1.462	1.554	1.646

5. Conclusions and Recommendations

The new generalization $\hat{C}''_{pk}$ has been proposed and was shown to be superior to other existing generalizations of C_{pk} for processes with asymmetric tolerances. Fortunately, explicit forms of the cumulative distribution function and the probability density function for the natural estimator $\hat{C}''_{pk}$, under the assumption of normality are also derived. In this paper, we perform the extensive calculations of the lower confidence bound on index C''_{pk} for process with asymmetric tolerance for various α-risks, $\hat{C}''_{pk}$, and sample sizes. For the conveniences of in-plant applications, we tabulate the lower confidence bounds for the purpose of decision making to test whether the processes with asymmetric tolerance is capable or not. In this paper, we also extensively discuss the relationships between lower confidence bound and parameter ξ for processes with all asymmetric tolerance and tabulate for the convenience for in-plant purposes.

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