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Designing a New Two-plan Sampling System by Variables with Switching Rules

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Abstract

Acceptance sampling plans contain the information about the sample size required for inspection and decision criteria for accepting or rejecting a lot using statistical principles. This paper attempts to develop a new variables two-plan sampling system, Tightened-Normal-Tightened (TNT), based on the process yield when the quality characteristic of the product has bilateral specification limits and follows a normal distribution. The plan parameters are determined by solving an optimization problem, which is designed to minimize the average sample number (ASN) and two main constraints are set such that the specified quality levels and risks can be satisfied simultaneously. The performance of the proposed system is studied and compared with the conventional variables single sampling plan. For practical purpose, the plan parameters under various quality requirements and risks are tabulated for practitioners to determine sample size and the associated decision rules for product acceptance determination.

Key words: *Consumer's risk, decision making, operating characteristic curve, process yield, producer's risk, quality assurance.*
JEL Classification: *C 44, C 02, C 46*

1. Introduction

Acceptance sampling plans contain the information about the sample size required for inspection and decision criteria for accepting or rejecting a lot, based on specified quality levels and allowable risks given by both producers and consumers using statistical principles.

There are several acceptance sampling plans with different strategies. The common way to classify acceptance sampling plans is by data type, i.e., attributes and variables. The main advantage of variables sampling plan is that more information can be obtained and the expected sample size required for inspection will be smaller than doing sampling plan by attributes (Montgomery, 2009; Schilling and Neubauer 2008). A new sampling system, named tightened-normal-tightened (TNT) sampling system, was investigated by Calvin (1977), Dodge (1965) and Hald and Thyregod (1965) for attribute data. The TNT sampling system involves two sampling plans with switching rules for achieving the desired operational characteristic (OC) curve. The procedure of TNT sampling system is similar to the well-known method, MIL-STD 105E but partially considers the tightened and normal inspections.

Muthuraj and Senthilumar (2006) and Senthilumar and Muthuraj (2010) extended the TNT sampling system to variables inspection. They concluded that the variables TNT sampling system can significantly reduce the required sample size but also provide the same protection. Hence, this paper attempts to develop a new variables TNT sampling system of type $\text{VTNT}(k_N, k_T, n)$ based on the process yield when the quality characteristic has bilateral specification limits and follows a normal distribution. The performance of the proposed sampling system is studied and compared to the conventional variables single sampling plan. Finally, the plan parameters are tabulated under various quality requirements for users to determine the required sample size and corresponding critical value.

2. Process yield index

Recently, process yield has been considered as an important criterion in the manufacturing industry for measuring process performance. In order to obtain an exact measure of process yield, Boyles (1994) proposed a process yield index, S_{pk}, for normally distributed processes with bilateral specification limits. The definition of S_{pk} is:

$$S_{pk} = \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\Phi\left(\frac{USL-\mu}{\sigma}\right)+\frac{1}{2}\Phi\left(\frac{\mu-LSL}{\sigma}\right)\right\}$$

Where USL and LSL represent the upper and lower specification limits, respectively, μ and σ are the mean and the standard deviation of the quality characteristic.

If $S_{pk} = S_0$, then the process yield can be expressed as $\%\,Yield = 2\Phi(3S_0)-1$, and the relationship between the number of nonconformities (NC) in unit of parts per million (PPM) and the S_{pk} index can be expressed as $\text{NC} = 2[1-\Phi(3S_0)]\times 10^6\,\text{PPM}$ or

$S_0 = \Phi^{-1}[1-(\text{NCPPM}\times 10^{-6}/2)]/3$. Obviously, there is one to one relationship between the process yield and S_{pk} value. For instance, if *%Yield* = 99.9900% or NC is 100 PPM, then the corresponding S_{pk} value is approximately to 1.2969.

For estimating the yield index S_{pk}, we consider the estimator $\hat{S}_{pk}$ which is calculated based on the collected sample data $x_1, x_2, \ldots, x_n$ from a stable process. The $\hat{S}_{pk}$ can be calculated by:

$$\hat{S}_{pk} = \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\Phi\left(\frac{USL-\bar{x}}{s}\right)+\frac{1}{2}\Phi\left(\frac{\bar{x}-LSL}{s}\right)\right\},$$

Where $\bar{x} = (\sum_{i=1}^{n} x_i)/n$ and $s = [(n-1)^{-1}\sum_{i=1}^{n}(x_i-\bar{x})^2]^{1/2}$ are the sample mean and sample standard deviation. Unfortunately, the sampling distribution of $\hat{S}_{pk}$ is a complex function and intractable to derive. Therefore, Lee *et al*. (2002) obtained a normal approximation distribution by Taylor's expansion with the first order. Hence, the sampling distribution of the estimator $\hat{S}_{pk}$ can be approximately distributed as:

$$\hat{S}_{pk} \xrightarrow{a} N(S_{pk}, \frac{a^2+b^2}{36n[\phi(3S_{pk})^2]})$$

Where *a* and *b* are function of μ and σ which can be defined as follows:

$$a = \frac{1}{\sqrt{2}}\left\{\frac{USL-\mu}{\sigma}\phi\left(\frac{USL-\mu}{\sigma}\right)+\frac{\mu-LSL}{\sigma}\phi\left(\frac{\mu-LSL}{\sigma}\right)\right\}.$$

$$b = \phi\left(\frac{USL-\mu}{\sigma}\right)-\phi\left(\frac{\mu-LSL}{\sigma}\right) = \phi\left(\frac{d-(\mu-M)}{\sigma}\right)-\phi\left(\frac{d+(\mu-M)}{\sigma}\right).$$

Lee *et al.* (2002) emphasized that even the size of *n* is smaller than 100, the approximation is still reasonable for practical purposes.

3. The proposed $\text{VTNT}(k_N, k_T, n)$ sampling system

The procedure of the proposed $\text{VTNT}(k_N, k_T, n)$ sampling system can be described as:

Step1: Start with tightened inspection, with the required sample size *n* and the corresponding critical value k_T.

Step2: When *t* lots in a row are accepted under tightened inspection, the next submission will be inspected under normal inspection, with the required sample size *n* and the corresponding critical value k_N $(< k_T)$.

Step3: When an additional lot is rejected in the next *s* lots after a rejection, the sampling procedure switches to tightened inspection (i.e. go back to STEP 1).

The probability of accepting the lot under tightened condition $P_T(p)$ and under normal condition $P_N(p)$ when the number of nonconformities is equal to *p* PPM can be calculated as follows respectively:

$$P_T(p) = P_a(\hat{S}_{pk} \ge k_T \,|\, p),$$

$$P_N(p) = P_a(\hat{S}_{pk} \ge k_N \,|\, p).$$

According to the concept introduced by Calvin (1977), the OC function of the proposed TNT sampling plan is given by:

$$\pi_A(p) = \frac{P_T(p)\cdot G + P_N(p)\cdot H}{G+H}$$

Where

$$G = \left\{[1-P_N(p)]^s\right\}\left\{[1-P_T(p)]^t\right\}[1-P_N(p)],$$

$$H = [P_T(p)]^t\,[1-P_T(p)]\left\{2-[P_N(p)]^s\right\}.$$

In order to design a variables TNT sampling system with a specified OC curve, one needs two designated points (p_{AQL} , $1-\alpha$) and (p_{LQL} , β). That is, the probability of acceptance should be at least $1-\alpha$ if the lot quality is at p_{AQL} PPM and the probability of acceptance should be no more than β if the lot quality is at p_{LQL} PPM. Thus, the plan parameters k_T , k_N and n can satisfy the following two nonlinear constraints simultaneously.

$$\pi_A(p_{\text{AQL}}) \ge 1-\alpha,$$

$$\pi_A(p_{\text{LQL}}) \le \beta.$$

It is desirable to have minimal sample size required for inspection. Let average sample number (ASN) is the expected number of sampled units required for lot determination, and then the plan parameters can be determined by solving an optimization model where the objective function is designed to minimize ASN with two nonlinear constraints mentioned above. That is,

Min ASN=*n*

Subject to

$$(1-\alpha) - \pi_A(p_{\text{AQL}}) \le 0,$$

$$\pi_A(p_{\text{LQL}}) - \beta \le 0,$$

$$n \ge 2,\; 0 < k_N < k_T.$$

4. The solved plan parameters

The solved plan parameters with selected quality requirements and risks are tabulated in Table 1. Based on the given table, the users can determine the required sample size and the corresponding critical values for lot sentencing.

Table 1: The plan parameters of the proposed method

p_{AQL}	p_{LQL}	$\alpha = 0.01$			$\alpha = 0.05$		
		$\beta = 0.05$			$\beta = 0.10$		
		k_T	k_N	n	k_T	k_N	n
100	500	1.2481	1.1603	247	1.2558	1.1603	135
	1000	1.2186	1.0968	115	1.2296	1.0968	63
	2000	1.1827	1.0301	65	1.1970	1.0301	35
500	1000	1.1399	1.0968	919	1.1435	1.0968	505
	2000	1.1132	1.0301	218	1.1205	1.0301	119
1000	2000	1.0751	1.0301	740	1.0789	1.0301	407

For instance, in the business contract, the allowable quality of submission of AQL is set to 100 PPM and the allowable quality of submission of LQL is set to 1000 PPM. Furthermore, both risks suffered from the producer and the consumer are set to $\alpha = 0.05$ and $\beta = 0.10$. The plan parameters k_N, k_T and n can be found in Table 1 as (1.0968, 1.2296, 63). It implies that a sample of size 63 should be randomly selected from the submitted lot and the lot should be accepted if the estimator $\hat{S}_{pk}$ > k_T = 1.2296 under tightened inspection; Otherwise, reject the lot if $\hat{S}_{pk} < 1.2296$. The system will be switched to normal inspection for the following submissions on condition that five lots in a row are successively accepted. Under normal inspection, the lot will be accepted if $\hat{S}_{pk}$ > k_N = 1.0968; Otherwise, reject the lot if $\hat{S}_{pk} < 1.0968$. And if an additional lot is rejected in the next four lots after rejection, switch to tightened inspection for the further judgment.

From Table 2, we can see that the required sample size for inspection of the proposed variables TNT sampling system is smaller than conventional variables single sampling (VSS) plan proposed by Wu and Liu (2014). The result points out that the proposed sampling system is economically superior to the VSS plan. This also implies that the proposed sampling system can guarantee the same protection for producer and consumer but with smaller required sample size so that the inspection cost will greatly be reduced. Hence, we may conclude that the proposed sampling system is an effective sampling strategy for lot sentencing.

Table 2: The required sample size for conventional VSS plan and the proposed variables TNT sampling system based on same requirements

p_{AQL}	p_{LQL}	$\alpha = 0.01$			$\alpha = 0.05$		
		$\beta = 0.05$			$\beta = 0.10$		
		VSS	TNT	Percentage of reduction	VSS	TNT	Percentage of reduction
100	500	650	247	62.00%	352	135	61.65%
	1000	291	115	60.48%	157	63	59.87%
	5000	80	36	55.00%	43	19	55.81%
500	1000	2522	919	63.56%	1366	505	63.03%
	5000	179	73	59.22%	96	40	58.33%
1000	5000	323	127	60.68%	174	69	60.34%

5. Conclusion

In this paper, a new variables TNT sampling system based on the process yield index is developed and compared with traditional variable single sampling plan. The results show that the proposed method requires fewer sample size for inspection but keep the same risk protection and quality requirements. Therefore, the proposed variables TNT sampling system can be recommended especially when the inspection is costly or destructive. In addition, the step-by-step procedure of the proposed sampling system is presented and tables of plan parameters are provided for practical purpose.

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