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An Integrated Replenishment Model for Transportation and Quantity Discounts Problem

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Abstract

Supply chain and inventory management is becoming more and more complex in today's production environment. To tackle the problem, different kinds of mathematical models have been proposed under different scenarios. This paper considers a joint replenishment problem (JRP) with batch transportation and quantity discounts. The problem is first formulated as a mixed integer programming (MIP) model with an objective to minimize total costs, which include ordering cost, holding cost, purchase cost and transportation cost, under a prerequisite that inventory shortage is prohibited in the system. Genetic algorithm (GA) is adopted next to solve the problem when it becomes too complicated to solve by the MIP. The GA has an attribute to find solutions that are very close to the optimal ones in a short computational time. Thus, the GA model can be very effective in searching for solutions, and it can be very useful for managers in real practice. A case study of a bike manufacturer is presented to examine the practicality of the models. The proposed method can provide a desirable decision making mechanism for the management.

Key words: *Joint replenishment problem (JRP), batch transportation, quantity discount, mixed integer programming (MIP), genetic algorithm (GA).*
JEL Classification: *C 60, C 61, M11*

1. Introduction

The joint replenishment problem (JRP) deals with a distribution of multiple items being purchased from a single supplier. Because a lower total cost for a firm often can be achieved when the right quantities of products are ordered from the same supplier, the JRP has become a popular topic in both the academic field and in real practice. The reasons for a lower total cost are basically two-fold. First, a larger quantity discount can often be obtained from a same supplier when the purchased amount is greater (Hsu, 2009; Wang et al., 2013b). Second, the combination of several items into a single purchase order can decrease the total fixed ordering costs. Usually, there are major fixed major ordering costs for placing an order, and also a minor ordering cost for each item purchased. When the fixed major ordering cost is relatively high, the bundling of different items into an order can reduce the total cost (Wang and Cheng, 2008; Wang et al., 2013b). Since the early works by Starr and Miller (1962) and Shu (1971), the JRP has been researched extensively.

GA, a heuristic search process for optimization, was first developed by Holland (1975), and it is a stochastic solution search procedure to solve combinatorial problems using the concept of evolutionary computation and imitating the natural selection and biological reproduction of animal species (Gen and Cheng, 2000). The GA has been widely applied to solve operations management, production, distribution, and inventory lot-sizing problems (Aytug et al., 2003; Gen and Syarif, 2005; Taleizadeh et al., 2011).

2. Model formulation

In this study, a MIP model is proposed first to solve the JRP with dynamic demands, quantity discounts and transportation cost. The goal is to minimize the total costs, which comprise of ordering cost, holding cost, purchase cost, and transportation cost, in a planning horizon and to set an appropriate inventory level for each part in each period.

The ordering cost for the system include the major ordering cost and the minor ordering cost, as shown in equation (1), where $\hat{o}$ is the major ordering cost per time, Y_t represents whether a purchase is made in period t, o_j is the minor ordering cost per time from supplier j, and X_{jt} represents whether a purchase is made from supplier j in period t.

$$\mathit{Ordering\ \ cost} = O = \hat{o} \times \sum_{t=1}^{T} Y_t + \sum_{t=1}^{T} \sum_{j=1}^{J} o_j \times X_{jt} \tag{1}$$

The ending inventory in a period is equal to the ending inventory level in the previous period plus the purchase quantity in the period minus the demand in the period. The holding cost in period t is defined to be equal to the holding cost per unit times the ending inventory in period t. Therefore, the ending inventory of part i in period t, I_{it}, is obtained by adding the beginning inventory of part i for period t (I_{it-1}) and the purchase quantity of part i in period t

(Q_{it}) and minus the demand of part i in the period t (d_{it}). Equation (2) shows the total holding cost for a planning horizon by summing up the holding cost for each part for each period

$$Holding\ \ cost\ = H = \sum_{t=1}^{T}\sum_{i=1}^{I} h_i \times I_{it} = \sum_{t=1}^{T}\sum_{i=1}^{I} h_i \times (I_{it-1} + \sum_{j=1}^{J} Q_{ijt} \times Z_{ijt} - d_{it}) \qquad (2)$$

The purchase cost is shown in Equation (3), where $P(Q_{ijt})$ is the unit purchase cost based on the discount schedule with the order quantity Q_{ijt}, and Z_{ijt} represents whether a purchased is made for part i from supplier j in period t.

$$Purchase\ \ cost = P = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T} (P(Q_{ijt}) \times Q_{ijt} \times Z_{ijt}) \qquad (3)$$

The transportation cost of the system is calculated using equation (4), where r_j is the transportation cost per time from supplier j per period, s_j is the maximum transportation batch size from supplier j , $\lceil Q_{ijt} / s_j \rceil$ is the smallest integer greater than or equal to Q_{ijt} / s_j , W_{jt} is number of transportations from supplier j in period t.

$$Transportation\ \ cost = R = \sum_{j=1}^{J}\sum_{t=1}^{T} r_j \times \left\lceil \sum_{i=1}^{I} Q_{ijt} \Big/ s_j \right\rceil = \sum_{j=1}^{J}\sum_{t=1}^{T} r_j \times W_{jt} \qquad (4)$$

3. Proposed Genetic Algorithm (GA) Model

The GA is adopted next to solve the JRP, and it can obtain near-optimal solutions in a short period of computational time when the problem becomes too complicated. The proposed procedures are as follows (Zegordi et al. 2010; Lee et al. 2013a; Lee et al. 2013b; Lu et al. 2013):

Step1: Code scheme.

Step2: Initial population of chromosomes.

Step3: Fitness function

Step4: Crossover operation

Step5: Mutation operator.

Step6: Selection of subsequent population.

Step7: Elitism selection.

Step8: Termination.

4. A Case Study of a Bicycle Manufacturing Company in Taiwan

A bicycle manufacturing company in Taiwan purchases some mechanical parts from Japan and assembles different models of bicycles. Finished bicycles are delivered to distributors around the world. The company needs to buy three parts: handle bars (part 1), brakes (part 2) and tires (part 3). It currently has two suppliers: supplier A and supplier B. Both suppliers provide quantity discounts, and the discount schedules for different parts are shown in Tables 1-3. The JRP model aims to minimize the total cost in the system and to set the optimal purchase quantity of each item from each supplier in each period. The planning horizon contains three days, and each day is a period. Table 4 shows the demand of the parts in each period in the planning horizon.

Table 1: Discount schedule for handlebars (part 1)

Supplier	Price break (*k*)	Purchase quantity (*Q*)	Price per unit (*P*(Q))
A	1	0~200	$350
	2	201~400	$320
	3	401 or more	$250
B	1	0~130	$400
	2	131~230	$300
	3	231 or more	$200

Table 2: Discount schedule for brakes (part 2)

Supplier	Price break (*k*)	Purchase quantity (*Q*)	Price per unit (*P*(Q))
A	1	0~130	$430
	2	131~230	$400
	3	231 or more	$250
B	1	0~170	$380
	2	171~470	$280
	3	471~	$240

Table 3: Discount schedule for tires (part 3)

Supplier	Price break (*k*)	Purchase quantity (*Q*)	Price per unit (*P*(Q))
A	1	0~170	$320
	2	171~370	$220
	3	371~	$180
B	1	0~60	$300
	2	61~160	$250
	3	161~	$200

Table 4: Demand of parts in each period in a planning horizon

d_{it} \ t	1	2	3
d_{1t}	240	10	50
d_{2t}	280	30	
d_{3t}	200		

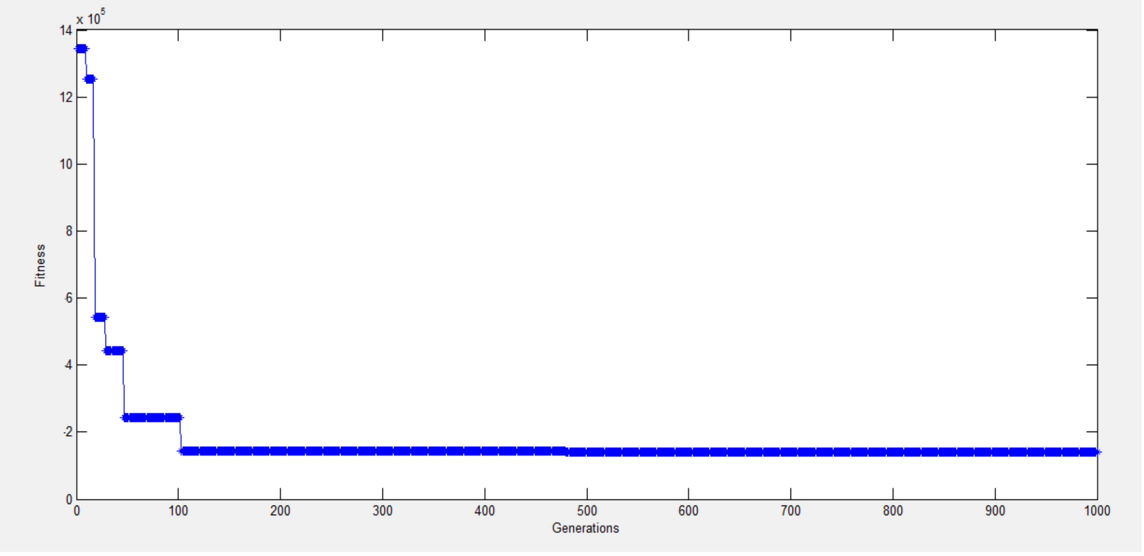
The proposed MIP model and the GA model are applied. The MIP model is solved by the software LINGO (2006), and the GA model is solved by the software MATLAB (2007). Two-cut-point crossover for crossover operator is used. An inversion mutation operator is applied to prevent a solution from being trapped in a local optimum and to help approach the global optimum. The initial population is 150. The crossover rate is set as 0.9, which means that around 90% pairs of individuals take part in the production of offspring. The mutation rate is set as 0.1, which indicates that each gene of a newly created solution is mutated with a probability of 0.1. The algorithm is terminated after 1000 generations.

The solutions obtained from the MIP model and from the GA algorithm are the same, and they are presented in Table 5. The replenishment policy is to purchase 300 units from supplier B for part 1, and 310 units from supplier A for part 2 in period 1. The total cost is \$141,620. Under the GA, the optimal solution is obtained at the 105th generation, as depicted in Figure 1.

Table 5: Solutions from the MIP and the GA model for the case

t	1	2	3	TC
Q_{11t}				
Q_{12t}	300			
Q_{21t}	310			
Q_{22t}				\$141,620
$P(Q_{11t})$				
$P(Q_{12t})$	\$200			
$P(Q_{21t})$	\$250			
$P(Q_{22t})$				

Figure 1: The convergence of GA for the case



5. Conclusions

Joint replenishment problem (JRP) has been a popular production management problem since the total cost for the firm may be reduced substantially if optimal quantities of different parts can be ordered from the same supplier at the same time. This paper develops a JRP model which considers batch transportation and quantity discounts. A mixed integer programming (MIP) model is constructed first to minimize total costs in a planning horizon, and a genetic algorithm (GA) model is developed next. The case study shows that both the MIP and the GA model can generate optimal solutions. For large-scale complicated problems that cannot be tackled by the MIP, the GA model shall generate near-optimal solutions under a short duration of time. This will be our future research direction. The results shall show that the GA model is an effective and efficient algorithm for managers in devising joint replenishment policies.

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