Resource Allocation with Undesirable Outputs: Evidence from Taiwan’s Container Shipping Liners

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Abstract

In the world, most international trade is carried out by shipping. Taiwan is surrounded by sea and is a key transportation hub in the Asia-Pacific region. Shipping industry in Taiwan plays an important role in international trade. In order to enhance the competitiveness of shipping companies, managers need to increase the overall efficiency. One way to achieve this is to allocate limited resource efficiently to different sub-units of an organization. When shipping companies provide the transportation service, the carbon emissions may be jointly produced by using various categories of resources. Some inputs might not be specifically allocated to sub-units, while others might be related to the production of carbon emissions. How to allocate common inputs to sub-units specifically and link potential carbon emission reductions with energy reductions are crucial issues in management of shipping companies. This paper proposes a novel centralized data envelopment analysis (CDEA) model, which consider both desirable and undesirable outputs in the allocation of a given resources, to explore the resource allocation among individual routes of a shipping company in Taiwan. To overcome some deficiencies of the traditional DEA model, we intend to propose a two-phase CDEA model. Based on the two-phase CDEA model, Russell directional distance function (RDDF) is adopted in Phase I model and the slack-based measure (SBM) is used in phase II to reallocate resource of common inputs, general inputs and carbon emission related inputs. Asia liner shipping routes operated by a Taiwanese container shipping liner for the year 2014 are investigated as an empirical study. The results provide valuable suggestions and managerial implications for management.

Key words: Container shipping liner, resource allocation, Centralized data envelopment analysis, Efficiency

JEL Classification: C 67, D 24, L 99
1. Introduction

More than 90% of international trade in the world is carried out by shipping (Hensher and Button, 2000). Taiwan is surrounded by sea and is a key transportation hub in the Asia-Pacific region. Shipping industry in Taiwan plays an important role in international trade. Hence, it is notable how to achieve the competitive advantage of shipping companies in Taiwan. In order to enhance or maintain the competitiveness, an organization that includes multiple units must seek out the highest possible overall efficiency by measuring individual unit efficiency and allocating resources among these sub-units (Giménez-García et al., 2007). A shipping company operates a number of individual routes, and a problem then arises with respect to how limited resources can be allocated in an optimal way to these routes to achieve the highest overall efficiency.

When shipping companies provide the transportation service, the carbon emissions may be jointly produced by using various categories of resources. Some inputs might not be specifically allocated to sub-units (e.g., empty position surcharge), hereafter common inputs, while others might be related to the production of carbon emissions (e.g., fuel cost), hereafter energy input. How to allocate common inputs to sub-units specifically and link potential carbon emission reductions with energy reductions are crucial issues in management of shipping companies.

Since Charnes et al. (1978) proposed the data envelopment analysis (DEA), it has become a powerful method for assessing efficiency. DEA is a non-parametric method that evaluates the relative efficiency of decision making units (DMUs) on the relationships between multiple inputs and multiple outputs. An advantage of DEA is that it does not require prior assumptions regarding relationships between inputs and outputs. It determine a piecewise linear surface by envelopment of the observed data. Hence, it can help the operators adjust their resources based on the empirical piecewise linear function. Although the original focus of this method is on the efficiency measurement, it has been revised and developed various DEA-based methods to deal with the resource allocation problem. Golany et al. (1993) proposed a target-setting model for input resource allocation at an organization level. Golany and Tamir (1995) introduced a DEA-based resource allocation model which set input and output targets by maximizing the sum of joint output. Athanassopoulos (1995, 1998) developed DEA models for resource allocation and target setting based on the goal programming technique. Beasley (2003) established a DEA model to jointly determine resource allocation and targets for each DMU. Amirteimoori and Tabar (2010) proposed a DEA-based approach for allocating fixed resource and deciding output target such that each DMU became efficient. Bi et al. (2011) introduced a DEA method for resource allocation in the parallel production system. Milioni et al. (2011) proposed a parametric DEA method to allocate resources. Hosseinzadeh Lotfi et al. (2013) proposed a common-weight DEA method.
for resource allocation and target setting. Wu et al. (2013) presented some new DEA-based resource allocation models. They considered both economic and environmental factors when a given resource was allocated. Du et al. (2014) used the cross-efficiency concept in DEA to solve the resource allocation problem.

In addition, Lozano and Villa (2004, 2005) and Lozano et al. (2004) introduced the centralized DEA (CDEA) models for resource allocation for the intra-organizational scenario. They focused on optimizing total resource consumption by all sub-units in an organization. Korhonen and Syrjänen (2004) introduced a multi-objective linear programming (MOLP) DEA approach in the concept of centralized resource allocation. Asmild et al. (2009) modified the centralized models proposed by Lozano et al. (2004) to only adjust inefficient units. Lozano et al. (2009) adopted a three-phase centralized DEA model to solve three objectives. These objectives include the maximum inflation of aggregated desirable production and the maximum deflation of input resources and undesirable total emissions. Lozano et al. (2011) established several non-radial CDEA models for resource allocation and target setting. Yu et al. (2013) proposed a modified CDEA model with a Russell measure to reallocate human resources. Since CDEA models use the centralized perspective instead of the individual perspective to allocate resource and maximize organizational performance, it is more suitable to investigate the problem of allocating limited resource in an organization. Hence, this paper constructs the resource allocation model based on the centralized DEA concept.

However most of above studies ignore undesirable outputs. Although Lozano et al. (2009) and Wu et al. (2013) provided methods for incorporating undesirable outputs into resource allocation models, how to links potential carbon emission reductions with energy reductions and allocates common inputs to sub-units are still ignored. In the shipping industry, carbon emissions, which are a kind of undesirable outputs, are non-separable with energy resource consumptions. Carbon emission reductions are proportional to the energy reductions. Since the study was proposed by Färe et al. (1989), modeling undesirable outputs in the DEA framework has been popular. In general, there are three cases of dealing with undesirable inputs. First, the hyperbolic measure approach (Färe et al., 1989; Zaim and Taskin, 2000a, b; Zofio and Prieto, 2001). Second, since an increase in undesirable outputs will incur DMUs’ costs, undesirable outputs are directly treated as inputs (Cropper and Oates, 1992). Third, the data transformation approach is used by integrates undesirable outputs into traditional DEA models (Zhu and Chen, 1993; Lovell et al., 1995; Seiford and Zhu, 2002, 2005; Zhu, 2003; Färe and Grosskopf, 2004; Yeh et al., 2010). In order to deal with this situation, the energy conservation performance index proposed by Guo et al. (2011) based on the weak disposability model in Färe et al. (1989) is applied, in which the energy inputs reductions are proportional to the decrease in carbon emission.

In addition, some inputs (general inputs) can be specifically allocated to sub-units, while
others (common inputs) are shared among different sub-units but not easy to allocate them specifically. Those common inputs cannot be separated. Hence, this difficulty in the allocation of common input resources should be considered when analyzing strategies to reasonably allocate resources. Moreover, since non-radial slacks are not considered in radial performance measurement, such as measurements with respect to Pareto inefficient projections, it is a good choice to use slacks-based measure in estimating those shortfalls or excesses of inputs needed to adjust among sub-units. In order to simultaneously solve non-proportional (non-radial) and proportional (radial) variables, this paper uses the Russell directional distance function (RDDF), which combines the concepts of the directional distance function and slacks-based measure, to allow changes of inputs and outputs in the proposed model.

The current paper proposes a novel CDEA model with two phases, which incorporates non-proportional and proportional variables into a unified model for the allocation of a given level of resources, to explore the resource allocation among Asia liner routes of a container shipping liner in Taiwan for the year 2014. Based on the two-phase CDEA model, RDDF is adopted in the first phase to obtain maximal total desirable outputs and minimal total undesirable outputs at current given level of resources, and SBM is used in the second phase to reallocate resources at optimal total desirable and undesirable outputs. The results provide valuable suggestions and managerial implications for management.

The contributions of this paper are fourfold: First, proposing a new resource allocation model that allows reallocation of resources by reduction or expansion of inputs among sub-units under a central authority. Second, linking potential carbon emission reductions with energy reductions. Third, incorporating both non-proportional and proportional variables into the proposed resource allocation model. Forth, separating input categories into common inputs, general inputs and energy inputs and allowing the allocation of common inputs among sub-units.

The rest of this paper is organized as follows. Section 2 develops models for resource allocation. In Section 3, the data are described and the empirical results are discussed. Finally, conclusions are drawn in Section 4.

2. Methodology

In order to simultaneously consider the existence of the desirable outputs, undesirable outputs, general inputs, common inputs and energy inputs, and solve the non-proportional adjustments of the desirable and undesirable outputs, we propose a two-phase CDEA model. In the first phase, the shipping company tries to maximize the production of the aggregated desirable outputs of all routes and minimize the production of the aggregated undesirable outputs of all routes at a given resource. In the second phase, given the optimal solution to the inflation rate of aggregated desirable outputs and the deflation rate of aggregated undesirable
outputs in the first phase, the shipping company tries to reallocate their resource. We assume that there are two scenarios for resource adjustment: minor adjustment and major adjustment. The minor adjustment assumes that the central authority can transfer common and general inputs among routes, and maintains the total number of common and general inputs as unchanged. The major adjustment assumes that common and general inputs can be reduced and transferred among routes.

Before describing the models, the notation should be introduced. Let $N$ be the number of routes; $I_1$ be the number of common inputs; $I_2$ be the number of general inputs; $I_3$ be the number of energy inputs; $R$ be the number of desirable outputs; $Q$ be the number of undesirable outputs. $j, k$ be the indexes for routes; $i_1$ be the index for common inputs; $i_2$ be the index for general inputs; $i_3$ be the index for energy inputs; $r$ be the index for desirable outputs; $q$ be the index for undesirable outputs; $x_{ij}$ be the amount of common input $i_1$ consumed by route $j$; $x_{ij}$ be the amount of energy input $i_3$ consumed by route $j$; $y_{rj}$ be the amount of desirable output $r$ produced by route $j$; $b_{qj}$ be the amount of undesirable output $q$ produced by route $j$; $\theta_r$ be the inflation of aggregated desirable output $r$; $\delta_q$ be the deflation of aggregated undesirable output $q$; $s_{i1k}$ be the slacks for common input $i_1$; $s_{i2k}$ be the slacks for general input $i_2$; $s_{i3k}$ be the slacks for energy input $i_3$; $\lambda_{ik1}$, $\lambda_{ik2}$, $\lambda_{ik3}$,...,$\lambda_{ikN}$ be the vector of the intensity variable for projecting route $k$; and $\alpha_{ij}$ be the shared rate of the common input quantities $x_{ij}$ for the route $j$.

The proposed two-phase CDEA model is written as follows:

**Phase I:**

The weighted-average inefficiency of each route can be estimated by solving the following DEA model based on the RDDF:

$$\begin{align*}
\text{Max} & \quad \frac{\sum_{r=1}^{R} \theta_r}{R} + \frac{\sum_{q=1}^{Q} \delta_q}{Q} \\
\text{s.t.} & \quad \sum_{j=1}^{N} \lambda_{ikj} x_{ij} \leq \alpha_{ik} x_{ik}, i_1 = 1, 2,..., I_1, i_2 = 1, 2,..., I_2, k = 1,..., N \quad (I-2) \\
\quad & \quad \sum_{j=1}^{N} \lambda_{ikj} x_{ij} \leq x_{ij}, i_2 = 1, 2,..., I_2, k = 1,..., N \quad (I-3) \\
\quad & \quad \sum_{j=1}^{N} \lambda_{ikj} x_{ij} \leq x_{ij}, i_3 = 1, 2,..., I_3, k = 1,..., N \quad (I-4) \\
\end{align*}$$

(Desirable output constraints)
\[
\sum_{k=1}^{N} \sum_{j=1}^{N} \lambda_{jk} y_{ij} \geq (1 + \theta_r) \sum_{k=1}^{N} y_{kj}, \quad r = 1, 2, \ldots, R
\]  
(I-5)

(Undesirable output constraints)

\[
\sum_{k=1}^{N} \sum_{j=1}^{N} \lambda_{jk} b_{ij} = (1 - \delta_q) \sum_{k=1}^{N} b_{jk}, \quad q = 1, 2, \ldots, Q
\]  
(I-6)

(Variable return-to-scale constraint)

\[
\sum_{j=1}^{N} \lambda_{jk} = 1, \quad k = 1, \ldots, N
\]  
(I-7)

(Shared rate constraints)

\[
\alpha_{ij} \leq \hat{\alpha}_{ij} \leq \tilde{\alpha}_{ij}, \quad i = 1, 2, \ldots, I_i, \quad j = 1, \ldots, N
\]  
(I-8)

\[
\sum_{j=1}^{N} \alpha_{ij} = 1, \quad i = 1, 2, \ldots, I_i
\]  
(I-9)

\[
\lambda_{jk} \geq 0, \quad j = 1, \ldots, N, \quad k = 1, \ldots, N
\]  
(I-10)

Where, \( \hat{\alpha}_{ij}, \tilde{\alpha}_{ij} \) are lower and upper bounds of shared rates of common inputs \( i = 1, 2, \ldots, I_i \) for routes \( j = 1, \ldots, N \) which are decided exogenously. Equation (I-1) pursues the optimum inflation of the aggregated desirable outputs and deflation of the aggregated undesirable outputs. Equation (I-2) ensures that the aggregated amount of each common input is no larger than the observed aggregated amount of each common input. Equations (I-3) and (I-4) imply that the general and energy input frontiers will be no larger than the observed general and energy inputs, respectively. Equation (I-5) pursues to non-proportionally increase the amount of each desirable output \( r \) as much as possible in the rate of \( \theta_r \), respectively, and ensures that each desirable output remains in the feasible aggregated output set. Equation (I-6) seeks to non-proportionally decrease the amount of each undesirable output \( q \) as much as possible in the rate of \( \delta_q \), respectively. Equation (I-7) shows that the assumption of variable return to scale is adopted in this model. In addition, it is necessary to specify a particular vector of lower and upper bounds of shared rates. Obviously, it depends on the context and the experience of the analyst in practice. For illustrative purposes, we consider the lower and upper bounds of shared rates are: \( (\hat{\alpha}_{ij}, \tilde{\alpha}_{ij}) = ((1-0.5) \frac{\hat{x}_{ij}}{x_q}, (1+0.5) \frac{\tilde{x}_{ij}}{x_q}) \).

Phase II:

There are two scenarios for resource adjustment. If the central authority wants to maintain the total number of common and general inputs as unchanged, the minor adjustment is adopted, while if the central authority wants to reduce all common and general inputs, the major adjustment is used. Situations when some inputs need to be increased to improve the performance are also likely to occur, the amount of allocable resources to be treated, should be increased rather than decreased as allowed in the proposed CDEA model. The slacks for
common input $i_1, s_{i_1k}$, can be separated into $s_{i_1k}^-$ and $s_{i_1k}^+$, where $s_{i_1k} = s_{i_1k}^- - s_{i_1k}^+$; the slacks for general input $i_2, s_{i_2k}$, can be separated into $s_{i_2k}^-$ and $s_{i_2k}^+$, where $s_{i_2k} = s_{i_2k}^- - s_{i_2k}^+$.

First, the SBM model for the minor adjustment is described as follows:

**Minor adjustment**

$$\text{Max} \sum_{i_1=1}^{i_1} \sum_{k=1}^{k} (s_{i_1k}^- - s_{i_1k}^+) + \sum_{i_2=1}^{i_2} \sum_{k=1}^{k} (s_{i_2k}^- - s_{i_2k}^+)$$

s.t.

(Common input constraints)

$$\sum_{j=1}^{j} \lambda_{jk} x_{i_1j} \leq \alpha_{i_1k} x_{i_1k} - s_{i_1k}^- + s_{i_1k}^+, i_1=1,2,\ldots,I_1, k=1,\ldots,N$$

$$\sum_{k=1}^{k} s_{i_1k} = \sum_{k=1}^{k} s_{i_1k}^+, i_1=1,2,\ldots,I_1$$

(General input constraints)

$$\sum_{j=1}^{j} \lambda_{jk} x_{i_2j} \leq x_{i_2k} - s_{i_2k}^- + s_{i_2k}^+, i_2=1,2,\ldots,I_2, k=1,\ldots,N$$

$$\sum_{k=1}^{k} s_{i_2k} = \sum_{k=1}^{k} s_{i_2k}^+, i_2=1,2,\ldots,I_2$$

(Energy input constraints)

$$\sum_{j=1}^{j} \lambda_{j} x_{i_3j} \leq (1 - \delta_{i_3k}^+) x_{i_3k}, i_3=1,2,\ldots,I_3, k=1,\ldots,N$$

(Desirable output constraints)

$$\sum_{k=1}^{k} \lambda_{jk} y_{rq} \geq (1 + \theta_{j}^+) \sum_{k=1}^{k} y_{rk}, r=1,2,\ldots,R$$

(Undesirable output constraints)

$$\sum_{k=1}^{k} \lambda_{jk} b_{pq} \geq (1 - \delta_{q}^+) \sum_{k=1}^{k} b_{pk}, q=1,2,\ldots,Q$$

(Variant return-to-scale constraint)

$$\sum_{j=1}^{j} \lambda_{jk} = 1, \ k=1,\ldots,N$$

$$\lambda_{jk} \geq 0, \ j=1,\ldots,N, k=1,\ldots,N$$

$s_{i_1k}, s_{i_2k}, s_{i_1k}^+, s_{i_2k}^+; \ free \ in \ sign, \ i_1=1,\ldots,I_1, i_2=1,\ldots,I_2, k=1,\ldots,N$
Where, $\theta_r^*(r = 1, \cdots, R)$, $\delta_q^*(q = 1, \cdots, Q)$ and $\alpha_{i,k}^*(i = 1, \cdots, I, k = 1, \cdots, N)$ are obtained in Phase I. Equation (II-1) indicates that the shipping company seeks to maximize the reduction ratios of common and general inputs. Equation (II-2) determines whether the current aggregated number of individual common inputs is appropriate. Equation (II-4) implies that the general input frontiers will be no larger than the observed general inputs. Equations (II-3) and (II-5) guarantee that the aggregated amount of individual common and general inputs should equal the original values. Equation (II-6) implies that undesirable output reductions are proportional to the energy input reductions. Undesirable outputs are non-separable with energy resource consumptions. Equations (II-7) and (II-8) can be seen as using the reallocation method to reach the $r$th ideal desirable output and the $q$th ideal undesirable output under the CDEA perspective.

**Major adjustment**

\[
\text{Max} \quad J = \sum_{i=1}^{I} \sum_{k=1}^{N} \left( s_{ijk} - s_{ijk}^* \right) x_{ijk} + \sum_{i=1}^{I} \sum_{k=1}^{N} \left( s_{ijk}^* - s_{ijk}^{**} \right) x_{ijk}^*
\]

s.t.

**(Common input constraints)**

\[
\sum_{j=1}^{J} \alpha_{i,j}^* x_{j} \leq \alpha_{i,j}^* x_{j} - s_{ijk}^* + s_{ijk}^{**}, i = 1, 2, \ldots, I, k = 1, \ldots, N
\]

**(General input constraints)**

\[
\sum_{j=1}^{J} \alpha_{i,j}^* x_{j} \leq x_{ijk} - s_{ijk}^* + s_{ijk}^{**}, i = 1, 2, \ldots, I, k = 1, \ldots, N
\]

**(Energy input constraints)**

\[
\sum_{j=1}^{J} \alpha_{i,j}^* x_{j} \leq (1 - \delta_{i,j}^*) x_{ijk}, i = 1, 2, \ldots, I, k = 1, \ldots, N
\]

**(Desirable output constraints)**

\[
\sum_{k=1}^{K} \sum_{j=1}^{J} \beta_{i,j} y_{jk} \geq (1 + \theta_i^*) \sum_{k=1}^{K} y_{jk}, r = 1, 2, \ldots, R
\]

**(Undesirable output constraints)**

\[
\sum_{k=1}^{K} \sum_{j=1}^{J} \beta_{i,j} b_{kj} \geq (1 - \delta_q^*) \sum_{k=1}^{K} b_{kj}, q = 1, 2, \ldots, Q
\]

**(Variable return-to-scale constraint)**
\[ \sum_{j=1}^{N} \lambda_{jk} = 1, \quad k = 1, \ldots, N \]  
(III-9)

\[ \lambda_{jk} \geq 0, \quad j = 1, \ldots, N, \quad k = 1, \ldots, N \]  
(III-10)

\[ s_{ijk}^x, s_{ijk}^y, s_{ijk}^z \geq 0, \quad i_1 = 1, \ldots, I_1, \quad i_2 = 1, \ldots, I_2, \quad k = 1, \ldots, N \]  
(III-11)

This model for the major adjustment policy is similar to the model for the minor adjustment policy. Equations (III-2), (III-4) and (III-6)-(III-9) can be compared to Equations (II-2), (II-4) and (II-6)-(II-9), respectively. Equations (III-3) and (III-5) ensure that the aggregated amount of individual common and general inputs should be less than their original value after resource reallocation.

However, the model in Phase I is non-linear. It should be transformed into a linear programming structure. Since the transformation process is complex, we first simplify the model to consider the situation that only includes one common input in the model, and then extend the model to consider the situation that includes two or more inputs in the model. The process is shown as follows:

**One common input model**

For each route \( k \), \( \alpha_{jk} \) is restricted to \( \alpha_{jk} = \alpha_j, \quad k = 1, \ldots, N \), \( j = 1, \ldots, N \) for constructing the piecewise frontier based on shared common input \( x_{ij} = \alpha_{j} x_i, \quad j = 1, \ldots, N \). Equation (I-2) then becomes Equation (I-2-1),

\[ \sum_{j=1}^{N} \lambda_{jk} \alpha_j x_i \leq \alpha_k x_i, \quad k = 1, \ldots, N . \]  
(I-2-1)

Since (I-2-1) is a nonlinear constraint, we need to linearize it. Let \( \lambda_{jk} \alpha_j = v_{jk} \), \( \lambda_{jk} = \mu_{jk} + v_{jk} \), \( v_{jk} = (1 - \alpha_j) \lambda_{jk} \), \( j = 1, \ldots, N \), \( k = 1, \ldots, N \)

Then, Phase I becomes

\[ \text{Max} \sum_{r=1}^{R} \delta_r + \sum_{q=1}^{Q} \delta_q \]  
\[ \text{s.t.} \]  
\[ \text{(Common input constraint)} \]

\[ \sum_{j=1}^{N} \mu_{jk} x_i \leq \alpha_k x_i, \quad k = 1, \ldots, N \]  
(I-2-2)

\[ \text{(General input constraints)} \]

\[ \sum_{j=1}^{N} (\mu_{jk} + v_{jk}) x_{ij} \leq x_{ik}, \quad i_1 = 1, 2, \ldots, I_1, \quad i_2 = 1, \ldots, I_2, \quad k = 1, \ldots, N \]  
(I-2-3)

\[ \text{(Energy input constraints)} \]

\[ \sum_{j=1}^{N} (\mu_{jk} + v_{jk}) x_{ij} \leq x_{ik}, \quad i_1 = 1, 2, \ldots, I_1, \quad k = 1, \ldots, N \]  
(I-2-4)
(Desirable output constraints)
\[
\sum_{k=1}^{N} \sum_{j=1}^{N} (\mu_{jk} + v_{jk}) y_{ij} \geq (1 + \theta_j) \sum_{k=1}^{N} y_{ijk}, \quad r = 1, 2, \ldots, R
\]
(I'-5)

(Undesirable output constraints)
\[
\sum_{k=1}^{N} \sum_{j=1}^{N} (\mu_{jk} + v_{jk}) b_{ij} = (1 - \delta_q) \sum_{k=1}^{N} b_{ik}, \quad q = 1, 2, \ldots, Q
\]
(I'-6)

(Variable return-to-scale constraint)
\[
\sum_{i=1}^{N} (\mu_{ik} + v_{ik}) = 1, \quad k = 1, \ldots, N
\]
(I'-7)

(Shared rate constraints)
\[
\tilde{\alpha}_j \leq \alpha_j \leq \tilde{\alpha}_j, \quad j = 1, \ldots, N
\]
(I'-8)

\[
\sum_{j=1}^{N} \alpha_j = 1
\]
(I'-9)

\[
\mu_{jk}, v_{jk} \geq 0, \quad j = 1, \ldots, N, \quad k = 1, \ldots, N
\]
(I'-10)

Multiplication of \( \mu_{jk} + v_{jk} \) on (I'-8), it becomes
\[
\tilde{\alpha}_j (\mu_{jk} + v_{jk}) \leq \alpha_j \leq \tilde{\alpha}_j (\mu_{jk} + v_{jk}), \quad j = 1, \ldots, N, \quad k = 1, \ldots, N
\]
(I'-8-1)

Replace (I'-8) by (I'-8-1), Phase I becomes a linear programming model.

Two or more common inputs model
Since \( \lambda_{jk} \alpha_{i,j} = \mu_{i,j,k}, \lambda_{jk} = \mu_{i,j,k} + v_{i,j,k}, \quad \alpha_{i,j} = (1 - \alpha_{i,j}) \lambda_{jk}, \quad i = 1, 2, \ldots, I_1, \quad j = 1, \ldots, N \), \( k = 1, \ldots, N \), since \( \lambda_{jk} = \mu_{i,j,k} + v_{i,j,k} \), we can arbitrarily select each of \( i'_i = 1, 2, \ldots, I_1 \) and impose \( \mu_{i,j,k} + v_{i,j,k} = \mu_{i,j,k} + v_{i,j,k}, \quad i = 2, \ldots, I_1, \quad j = 1, \ldots, N \), \( k = 1, \ldots, N \) on Phase I. Without loss of generality, let \( \mu_{jk} + v_{jk} = \mu_{i,j,k} + v_{i,j,k} \) to preserves the linearity and convexity in the DEA model.

Then, Phase I becomes
\[
\text{Max} \quad \frac{\sum_{j=1}^{R} \theta_j}{R} + \frac{\sum_{q=1}^{Q} \delta_q}{Q}
\]
(I'-1)

s.t.

(Common input constraints)
\[
\sum_{j=1}^{N} \mu_{i,j,k} x_{ij} \leq \alpha_{i,k} x_{i}, \quad i = 1, 2, \ldots, I_1, \quad k = 1, \ldots, N
\]
(I'-2)

(General input constraints)
\[
\sum_{j=1}^{N} (\mu_{jk} + v_{jk}) x_{ij} \leq x_{ik}, \quad i = 1, 2, \ldots, I_2, \quad k = 1, \ldots, N
\]
(I'-3)

(Energy input constraints)
\[
\sum_{j=1}^{N} (\mu_{jk} + v_{jk}) x_{ij} \leq x_{ik}, i = 1,2,\ldots, I, k = 1,\ldots, N \quad (1')-4
\]

(Desirable output constraints)

\[
\sum_{k=1}^{N} \sum_{j=1}^{N} (\mu_{jk} + v_{jk}) y_{ij} \geq (1 + \theta_{I}) \sum_{k=1}^{N} y_{jk}, r = 1,2,\ldots, R \quad (1')-5
\]

(Undesirable output constraints)

\[
\sum_{k=1}^{N} \sum_{j=1}^{N} (\mu_{jk} + v_{jk}) b_{ij} = (1 - \delta_{I}) \sum_{k=1}^{N} b_{kj}, q = 1,2,\ldots, Q \quad (1')-6
\]

(Versatile return-to-scale constraint)

\[
\sum_{j=1}^{N} (\mu_{jk} + v_{jk}) = 1, i = 1,2,\ldots, I, k = 1,\ldots, N \quad (1')-7
\]

(Shared rate constraints)

\[
\alpha_{ij} \leq \alpha_{i'j} \leq \alpha_{ij}, i = 1,2,\ldots, I, j = 1,\ldots, N \quad (1')-8
\]

\[
\sum_{j=1}^{N} \alpha_{ij} = 1, i = 1,2,\ldots, I \quad (1')-9
\]

\[
\mu_{ij} + v_{ij} = \mu_{i'j} + v_{i'j}, i = 2,\ldots, I, j = 1,\ldots, N \quad (1')-10
\]

Multiplication of \( \mu_{jk} + v_{jk} \) on \((1')-8\), it becomes

\[
\alpha_{ij} (\mu_{jk} + v_{jk}) \leq \alpha_{i'j} (\mu_{k'j} + v_{k'j}), k = 1,2,\ldots, I, j = 1,\ldots, N \quad (1')-8-1
\]

Replace \((1')-8\) by \((1')-8-1\), Phase I becomes a linear programming model.

3. Empirical results

3.1 Data and input-output variables

The data set used in this paper consists of 14 Asia liner shipping routes operated by a Taiwanese container shipping liner for the year 2014. Due to the confidentiality, the name of this container shipping liner cannot be provided. In order to allocate the resources of those 14 routes, the common inputs, general inputs, energy inputs, desirable outputs and undesirable outputs must be identified.

The common input variables include crew salary (CS) and empty position surcharge (EPS). The general inputs consist of ship capacity (SC), handling expense (HE) and other cost (OC). The energy input is fuel cost (FC). The desirable output is revenue (R). The undesirable output is carbon emission (CE). The descriptive statistics of the data are presented in Table 1.
Table 1: Descriptive statistics of inputs and outputs

<table>
<thead>
<tr>
<th>Total</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Max</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Common inputs</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CS</td>
<td>62.513</td>
<td>4.465</td>
<td>13.276</td>
<td>0.000</td>
</tr>
<tr>
<td>EPS</td>
<td>557.921</td>
<td>39.852</td>
<td>72.543</td>
<td>19.102</td>
</tr>
<tr>
<td><strong>General inputs</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SC</td>
<td>151,959.00</td>
<td>10,854.210</td>
<td>18,200.000</td>
<td>5,200.000</td>
</tr>
<tr>
<td>HE</td>
<td>2,282.834</td>
<td>163.060</td>
<td>288.958</td>
<td>64.720</td>
</tr>
<tr>
<td>OC</td>
<td>3,476.828</td>
<td>248.345</td>
<td>396.661</td>
<td>107.870</td>
</tr>
<tr>
<td><strong>Energy input</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FC</td>
<td>2,402.232</td>
<td>171.588</td>
<td>272.990</td>
<td>73.973</td>
</tr>
<tr>
<td><strong>Desirable output</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>7,720.72</td>
<td>551.480</td>
<td>1,028.525</td>
<td>261.708</td>
</tr>
<tr>
<td><strong>Undesirable output</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CE</td>
<td>75.087</td>
<td>5.363</td>
<td>15.823</td>
<td>1.731</td>
</tr>
</tbody>
</table>

3.2 Adjustment analysis

3.2.1 Phase I analysis

In the first phase, the optimal values of the inflation of desirable outputs and the deflation of undesirable outputs are obtained. The result indicates that the value of the inflation of revenue is equal to 0.078, and the value of the deflation of carbon emissions is equal to 0.313, meaning that the aggregated revenue of 14 routes can be increased from 7,720.726 to 8,322.943, and the aggregated carbon emissions of those routes can be decreased from 75.087 to 51.585. The distributions of the common inputs are presented in Table 2. The results indicate that Routes A, H and N should be assigned more common expense than other routes, and Routes A, B and H should be assigned more empty position surcharge than other routes. In particular, Route A should obtain about one-third of the total common expense and about a fifth of the empty position surcharge.

Table 2: The distribution of the common inputs of routes

<table>
<thead>
<tr>
<th>route</th>
<th>Share rate of CS</th>
<th>Share rate of EPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.316 (19.754)</td>
<td>0.195 (108.795)</td>
</tr>
<tr>
<td>B</td>
<td>0.069 (4.313)</td>
<td>0.114 (63.603)</td>
</tr>
<tr>
<td>C</td>
<td>0.007 (0.438)</td>
<td>0.064 (35.707)</td>
</tr>
<tr>
<td>D</td>
<td>0.067 (4.188)</td>
<td>0.036 (20.085)</td>
</tr>
<tr>
<td>E</td>
<td>0.038 (2.375)</td>
<td>0.063 (35.149)</td>
</tr>
<tr>
<td>F</td>
<td>0.048 (3.001)</td>
<td>0.076 (42.402)</td>
</tr>
<tr>
<td>G</td>
<td>0.071 (4.438)</td>
<td>0.044 (24.549)</td>
</tr>
</tbody>
</table>
3.2.2 Minor adjustment analysis

The minor adjustment strategy aims at reallocating the common and general inputs among routes, under the assumption that the aggregated amounts of individual common and general inputs remain unchanged. The results of the minor adjustment analysis are presented in Table 3.

Table 3 indicates that Route M show no change in all common and general inputs. Routes B, D, E, F, G, H, I, K and N can transfer their crew salaries to Routes A, C and J. Routes A, B and H can decrease their empty position surcharges, while Routes C, D, E, F, G, I, J, K, L and N can increase their empty position surcharges. Routes A, B, C, D, H, I, J, K and N can transfer their ship capacities to Routes E, G and L. Routes B, C, D, H, I, J and K can transfer their handling expenses to Routes A, E, F, G, L and N. Routes A, B, C, F, H, I, J, K and N can decrease their other costs, while Routes D, E and L can increase their other costs. In addition, the results of reduced quantity of fuel cost of individual routes are indicated in Table 3. The results show that the aggregated fuel costs of those routes can be decreased from 2,402.232 to 1,650.330. Routes C, E, F, J, L and M need to reduce fewer quantities of fuel cost that other routes, meaning that those routes have lower carbon emissions.

<table>
<thead>
<tr>
<th>Route</th>
<th>Slacks for CS</th>
<th>Slacks for EPS</th>
<th>Slacks for SC</th>
<th>Slacks for HE</th>
<th>Slacks for OC</th>
<th>Reduced quantity of FC</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>28.422</td>
<td>-32.295</td>
<td>-2,248.6</td>
<td>194.822</td>
<td>-115,203</td>
<td>-75,229</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>B</td>
<td>-3.079</td>
<td>-29.176</td>
<td>-3,804.3</td>
<td>-20.257</td>
<td>-17,842</td>
<td>-69,415</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>00</td>
</tr>
<tr>
<td>C</td>
<td>0.450</td>
<td>6.750</td>
<td>-3,570.0</td>
<td>-16,800</td>
<td>-52,100</td>
<td>-27,906</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>00</td>
</tr>
<tr>
<td>D</td>
<td>-2.953</td>
<td>14.781</td>
<td>-5,378.8</td>
<td>-23,423</td>
<td>12,008</td>
<td>-67,179</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>70</td>
</tr>
<tr>
<td>E</td>
<td>-1.500</td>
<td>7.200</td>
<td>400.000</td>
<td>1,000</td>
<td>16,500</td>
<td>-27,622</td>
</tr>
<tr>
<td>F</td>
<td>-2.100</td>
<td>0.300</td>
<td>0</td>
<td>5,100</td>
<td>-5,500</td>
<td>-23,154</td>
</tr>
</tbody>
</table>

Table 3: Results of minor adjustment strategy

Note: Amounts of allocated common inputs are provided in parentheses.
3.2.3 Major Adjustment Analysis

The major adjustment strategy aims at reallocating and reducing the common and general inputs among routes. The results of the major adjustment analysis are shown in Table 4.

Table 4 indicates that, the central authority could reduce the aggregated amount of the crew salary by 38.249, the aggregated amount of the empty position surcharge by 23.559, the aggregated amount of the ship capacity by 31.344.697, the aggregated amount of the handling expense by 33.186 and the aggregated amount of the other cost by 298.796 without affecting the production of the maximum desirable output and the minimum undesirable output. Route M shows no changes in all common and general inputs. Routes A, B and H suggest a need to decrease all common and general inputs. Routes C and J require an increase in both crew salary and empty position surcharge, but a decrease in ship capacity, handling expense and other cost. Route D suggests a need to increase crew salary, empty position surcharge, handling expense and other cost, but decrease ship capacity. Route E requires an increase in empty position surcharge, ship capacity, handling expense and other cost, but a decrease in crew salary. Route F suggests a need to increase ship capacity, handling expense and other cost, but reduce both crew salary and empty position surcharge. Routes G and N require an increase in both empty position surcharge and handling expense, but a decrease in crew salary, ship capacity and other cost. Routes I and K suggest a need to increase empty position surcharge, but reduce crew salary, ship capacity, handling expense and other cost. Route L requires an increase in empty position surcharge, ship capacity, handling expense and other cost. In terms of the reduced quantity of fuel cost, the optimal value of the deflation of undesirable outputs is obtained from Phase I, the reduced quantities of fuel cost of individual
routes in the major adjustment strategy are equal to those of individual routes in the minor adjustment strategy.

Table 4: Results of major adjustment strategy

<table>
<thead>
<tr>
<th>Route</th>
<th>Slacks for CS</th>
<th>Slacks for EPS</th>
<th>Slacks for SC</th>
<th>Slacks for HE</th>
<th>Slacks for OC</th>
<th>Reduced quantity of FC</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-18.619</td>
<td>-75.628</td>
<td>-880.857</td>
<td>-100.497</td>
<td>-113.785</td>
<td>-75.229</td>
</tr>
<tr>
<td>B</td>
<td>-3.079</td>
<td>-29.176</td>
<td>-3,804.3</td>
<td>-20.257</td>
<td>-17.842</td>
<td>-69.415</td>
</tr>
<tr>
<td>C</td>
<td>0.450</td>
<td>6.750</td>
<td>-3,570.0</td>
<td>-16.800</td>
<td>-52.100</td>
<td>-27.906</td>
</tr>
<tr>
<td>D</td>
<td>4.963</td>
<td>46.598</td>
<td>-4,875.8</td>
<td>22.407</td>
<td>64.591</td>
<td>-67.179</td>
</tr>
<tr>
<td>E</td>
<td>-1.500</td>
<td>7.200</td>
<td>400.000</td>
<td>1.000</td>
<td>16.500</td>
<td>-27.622</td>
</tr>
<tr>
<td>F</td>
<td>-1.400</td>
<td>-14850</td>
<td>6,893.00</td>
<td>174.500</td>
<td>223.200</td>
<td>-23.154</td>
</tr>
<tr>
<td>G</td>
<td>-3.013</td>
<td>6.520</td>
<td>-7,308.6</td>
<td>17.138</td>
<td>-96.741</td>
<td>-85.446</td>
</tr>
<tr>
<td>H</td>
<td>-5.319</td>
<td>-56.578</td>
<td>-2,180.8</td>
<td>-75.097</td>
<td>-95.585</td>
<td>-75.229</td>
</tr>
<tr>
<td>I</td>
<td>-0.329</td>
<td>7.124</td>
<td>-3,241.3</td>
<td>-79.657</td>
<td>-107.542</td>
<td>-69.415</td>
</tr>
<tr>
<td>J</td>
<td>0.750</td>
<td>28.300</td>
<td>-3,390.0</td>
<td>-24.400</td>
<td>-69.000</td>
<td>-27.906</td>
</tr>
<tr>
<td>K</td>
<td>-0.303</td>
<td>14.131</td>
<td>-5,378.8</td>
<td>-4.423</td>
<td>-38.292</td>
<td>-67.179</td>
</tr>
<tr>
<td>L</td>
<td>0</td>
<td>32.900</td>
<td>400.000</td>
<td>19.500</td>
<td>11.700</td>
<td>-27.622</td>
</tr>
<tr>
<td>M</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-23.154</td>
</tr>
<tr>
<td>N</td>
<td>-10.850</td>
<td>3.150</td>
<td>-4,407.0</td>
<td>53.400</td>
<td>-23.900</td>
<td>-85.446</td>
</tr>
<tr>
<td>T</td>
<td>-38.249</td>
<td>-23.559</td>
<td>-31,344.0</td>
<td>-33.186</td>
<td>-298.796</td>
<td>-751.902</td>
</tr>
<tr>
<td>Total</td>
<td>-38.249</td>
<td>-23.559</td>
<td>-31,344.0</td>
<td>-33.186</td>
<td>-298.796</td>
<td>-751.902</td>
</tr>
</tbody>
</table>

4. Conclusions

This paper proposes a new resource allocation model based on the CDEA to investigate the reduction and transformation of resource among 14 Asia liner shipping routes operated by a Taiwanese container shipping liner in 2014. In the proposed model, the inputs are divided into common, general and energy inputs, and the potential carbon emission reductions are linked with energy reductions. In addition, both non-proportional and proportional variables...
are incorporated into the proposed model. The empirical results show that the different scenarios provide different adjustment strategies. In order to improve the efficiency of the overall organization, the shipping company can choose the more moderate strategy which maintains the original levels of resources and transfers resources among routes, or the more sweeping strategy which reduces and transfers resources among routes.

However, the paper has some limitations. First, environmental factors are excluded in this paper, which might affect the resource allocation results. Second, the cost of transferring resources is ignored. Third, the internal structure of operational process is not considered. In future researches, the above-mentioned limitations can be investigated.

**References**


