Modeling Volatility: Indian Stock and Foreign Exchange Markets

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Abstract
This paper attempts to evaluate the ability of the different statistical models used for forecasting volatility in the context of Indian stock and foreign exchange markets. The financial market volatility has remained very calm from 2004 to late 2006 and registered below long-term averages as measured by VIX index. However, the financial crisis of 2008 changed the entire picture rendering markets extremely ‘volatile’. Therefore, this paper examines the properties of several models used in volatility forecasting; Historical volatility models (EWMA), ARCH and GARCH family models (TARCH, EGARCH, PARCH, etc.) and their accuracy in modeling and forecasting the volatility of Indian Rupee against USD and index return movements. The problem that this paper seeks to answer is identifying the ideal model which better explains the volatility and asymmetry of stock and foreign exchange markets. NSE and BSE index return series is used for examining the stock market volatility. In this direction, the paper is divided into two parts: model selection and comparison of various models on the basis of forecast error statistics. Based on the forecast statistics we find that TARCH and PARCH will lead to better volatility forecast for BSE and NSE return series for the stock market evaluation and ARMA (1,1)/ ARCH/ EGARCH for the foreign exchange market.

JEL Classification: G14, G15, G17
1. Introduction

Volatility refers to fluctuations observed in some process over time. In financial economics, volatility is the standard deviation of the Weiner-driven component within a continuous-time diffusion model. Volatility is very important when pricing the derivative securities, whose volume has increased manifold in the recent years. For this, we need to know the volatility of the underlying asset until the option expires. In fact, the market convention now lists option prices in terms of volatility units; one can buy derivatives written on volatility, in which case the exact volatility will be clearly specified in the derivative contracts. So, a volatility forecast is needed for such derivative contracts.

Moreover, risk management has become important since the establishment of first Basle Accord (1996). This makes volatility forecasting a necessary risk-management exercise for most financial institutions around the world. In addition to that, reserve capital of at least three times of value-at-risk (VaR) has to be set aside by banks and financial institutions, defined as the minimum expected loss with a 1% confidence level for a given time period. Such VaR values are readily available given volatility forecast, mean estimate, and normal distribution. Volatility in financial markets has a wide repercussion on the economy. The incidents of terrorists’ attack (September 11, 2001 and more), and the recent financial scandals have caused great impact on the financial markets. This evidences important relation between uncertainty of financial market and confidence of people. Therefore, policy makers rely on market volatility as a barometer for the vulnerability of the economy and financial markets which shows the importance of volatility forecasting. We have several models that can forecast volatility in a time series. Depending upon data availability as well as the intended use of the model, volatility models work upon either on discrete or continuous time. As we all know, that trading and prices of the securities is evolving in a near continuous trend. It is, however, useful in many situations to formulate the model in discrete time.

There are many reasons why modeling and forecasting volatility has been subject of many theoretical and practical studies, quite recently:

1. Volatility is the integral factor of derivative security pricing formula. Black-Scholes suggested an option pricing formula as a function of volatility.

2. Volatility index become one of the financial instruments in recent time. VIX volatility index started to trade in futures from 2004.

3. If investors base their calculations on mean-variance relationship for asset allocation, volatility plays vital role in helping them in the investment decision.
As a proxy of risk, volatility is not only of a great concern to investors but also to the policy makers. Direct impact of the time varying volatility on the pricing and hedging of derivatives, is what investors are interested in, whereas, policy makers are mainly interested in the effect of volatility on the stability of financial markets in particular and the stability of the whole economy in general. Finally, volatility forecasting is essential in many value-at-risk models. Because of its numerous applications in financial market, volatility is researched upon in plentiful studies for accurate forecast and measure.

Given the \( n \) number of models available for the empirical analysis and strategic evaluation of the above mentioned phenomenon, there exist no single criteria for analysis and hence this paper explores into the pros and cons of using different models and suggests the appropriate model for the Indian stock and foreign exchange market volatility evaluation.

The purpose of this study is to compare the different statistical methods used in volatility forecasting. It investigates out-of-sample predictive ability of the various models in the context of stock and foreign exchange markets. The data covers a period of considerable change in these markets. The period studied is 1990 to 2004 which spans the introduction of the Euro, the phase of several developing markets moving to open economies, and a significant depreciation on the US dollar. We tested several classes of widely used models: historical (including moving averages EWMA), autoregressive conditional heteroskedasticity models, GARCH family models (EGARCH, TARCH, PARCH)

Following this Section 1, Section 2 reviews the existing literature and the empirical findings of the various models. Here, we discuss about the relation between models and their differences as well. Section 3 deals with the data followed by Section 4 wherein different models used for measuring volatility have been discussed. The empirical findings have been discussed in Section 5. Results and discussions are given in Section 6 followed by the conclusions which is given in Section 7 which is the final and concluding section.

2. Literature review

The works of Mandelbrot (1963) and Fama (1965) were the first few works that examined the statistical properties of stock returns; Akgiray’s (1989) work proceeds further in the same direction which not only investigates the statistical properties but also presents evidence on the forecasting ability of ARCH and GARCH models vis-à-vis EWMA (exponentially weighted moving average) and the Historic simple average method. While Pagan and Schwert (1990) report that GARCH and EGARCH models including terms suggested by nonparametric methods yield significant increases in explanatory power: in the same year Dimson and Marsh (1990)
came up with rather interesting finding that simple models performed better than the exponential smoothing or regression based methods. Of course it has to be noted that their study does not include the popular ARCH family of models in contrast to this Tse (1991), Tse and Tung (1992) found that EWMA models provided better forecasts than the GARCH models. These studies were conducted in different markets – the former was carried in U K stock market while the later was examined in the Japanese and Singapore markets respectively. Franses and van Dijk (1996) examined the forecasting ability of the GARCH family of models against random walk model in five European stock markets and found that random walk model fares better even when the period of 1987 crash was included. Brailsford and Faff (1996) investigated the forecasting models in the Australian market and found that ARCH class of models and simple regression provide better forecasts but the rankings were sensitive to the error statistic used to assess the accuracy of the forecast. In the context of foreign exchange markets West and Cho (1995) find evidence in favor of GARCH model over shorter intervals and in the longer horizon no model fare better. Some of the more recent works were by Loudon et al (2000), Mcmillan et al (2000), Yu (2002), Klaassen (2002), Vilasuso (2002) and Balaban (2004).

In the Indian context Varma (1999) investigated the volatility estimation models making comparison between GARCH and the EWMA models in the risk management setting. Pandey in 2002 explored the extreme value estimators and found that they perform better than the traditional close to close estimators although his study does not consider the performance of extreme value estimators versus time varying volatility models. Kaur (2004) examined the nature and characteristics of stock market volatility in India. Significance of coming up with improved model since before crisis period has prompted Chou (2005) to argue that the failure of all the range-based models is due to the fact that temporal movements of price range has been ignored. Using a structure that includes conditional expectation of range, the Conditional Autoregressive Range models (Chou 2005), solves this puzzle. Similarly, (2006)Brandt and Jones formulated a model analogous to Nelson’s (1991) EGARCH, but uses the square root of the intra-day price-range instead of using absolute return. Guglielmo Mario, Juncal Cunado and Luis A. Gil-Alana (2013) modelled long-run cycles in financial time series. M.hashem and Andreas Pick (2010) Forecasted the Combination across Estimation Windows. Vafeadis, Bora and Dimitris (2011) also estimated trend in time series. Although attempts have been made to introduce new models so as to have more accurate forecast, only few literature available has dealt with the comparison of these model. Moreover, there is celebrated number of empirical findings; researchers so far have primarily concentrated on the developed markets, such as US and Europe. This paper, hence,
attempts a comparison between the mentioned arrays of model and concludes onto the suggested ones in the context of Indian markets.

3. Data description

In this paper we considered the NIFTY and SENSEX index as the proxy for the stock market and accordingly the closing index values were collected from March 31, 1994 – March 28, 2013 for NSE index and from Jan 01, 1990- May 21, 2013 for BSE index.

The exchange rate data was pertaining to the Indian rupee/US dollar exchange rate over the period Jan 03, 2000 till May 17, 2013 was used as a proxy for the foreign exchange market. Out of the total observations the data pertaining to

- 1994 till Dec 2004 for NIFTY index
- 1990 till Dec 2002 for SENSEX index
- 2000 till Dec 2006 for exchange rate data

Were used for estimation of the model parameters and the remaining observations were used for out of sample forecasting also known as hold out sample.

The data was collected from CMIE Prowess database and www.federalreserve.gov and www.rbi.gov. The daily observations were converted into continuous compounded returns in the standard method as the log differences:

\[ r_t = 100 \times \ln \left( \frac{I_t}{I_{t-1}} \right) \]

Where \( I_t \) stands for the closing index value/exchange rate on day \( t \).

Where \( r_t \) is the daily return on day \( t. \)

The data used includes the crisis period data which has the ability to give biased results as seen from unit root, Dicker-Fulley, ACF/PACF and other tests and hence needs to be corrected. The number of differences required to get a stationary series is determined by the “i” term in the ARIMA (p, i, q) model. Therefore, in this paper, we correct the abnormal from additive-outlier detection method found in GARCH models which was developed by Franses and Ghijsels (1999), it was extended to detection of innovative outliers by Charles-Darné (2005). We studied the effects of outlying observations and we show that the equation parameters are biased when we remove the innovative and additive outliers. This leads us to present that the volatility forecast is better when the data is corrected for outliers for several step-ahead forecasts even if we employ a GARCH-\( t \) process for short-term or long term forecasts.
The descriptive statistics of the data is presented in Table 1 and figures 1(a), 1(b) and 2 plots the return series of Nifty, Sensex and exchange rate respectively. All three series exhibit excess kurtosis indicating that the unconditional return distributions are not normally distributed. The Jarque-Bera (JB) statistic confirms that normality is rejected at a p-value of almost 1. From figures 3 and 4 we can note that the returns exhibit fat tails which is more prominent for the exchange rate series. The plot of return series in Figures 1 and 2 shows that volatility clustering is a feature of both the markets which suggests that the volatility is predictable. The Ljung-Box Q statistics for the return and squared return series show that the null hypothesis of no serial correlation can be rejected at 36th lag for both series. To test for possible unit roots the augmented Dickey-Fuller (ADF) statistic is calculated and the results show that the series is stationary. The null hypothesis of unit root can be rejected in all the cases at 5% level of significance.

See Table 1 &
Figures 1, 2, 3 and 4

The absence of unit root means that the series is stationary, combined with the phenomenon of volatility clustering implies that volatility can be predicted and the forecasting ability of various models can be generalized to other time periods also.

4. Competing models

4.1 EWMA MODEL

The EWMA (Exponentially Weighted Moving Average) model used in J. P. Morgan’s Risk Metrics® methodology does well at the 10% and 5% risk levels but breaks down at the 1% and lower risk levels. Exponential smoothing is an adaptive forecasting method that gives greater weight to more recent observations so that the finite memory of the market is represented. It is used to model variances including the weighted averages given for all points and the model equation is as follows:

\[ \sigma_t^2 = \sum_{i=1}^{m} \alpha_i R_{t-i}^2, \]

Where the weights \( \alpha_i \), decrease exponentially as we tend to move back in time, decrease exponentially as move back through time. Specifically, it turns out that this weighting scheme leads to a particularly simple formula for updating volatility estimates. The formula is
Solving the equation we get:

\[
\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda) R_{t-1}^2,
\]

The weights for the \( R_i \)'s keep on declining at rate \( \lambda \) as we move back through time. Each weight is \( \lambda \) times the previous weight. Therefore known as exponential weighted moving average.

4.2 ARCH model

The Autoregressive Conditional Heteroskedasticity (ARCH) model was first introduced and researched upon by Engle in 1982 (Engle, 1982). ARCH model and its extensions are among the most popular models for forecasting market returns and volatility. Originally, the ARCH model rather than using standard deviations used the variance. Let us call the variance of the returns \( \sigma^2 \) as \( h \). The ARCH model can be specified as follows:

\[
r_t = \mu + \varepsilon_t
\]
\[
\varepsilon_t = \sqrt{h_t} \varepsilon_t
\]
\[
h_t = \sigma^2 + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2
\]

where:

- \( r_t \) is the return at time \( t \)
- \( \mu \) - mean return
- \( \varepsilon_t \) - residuals (or error terms)
- \( \varepsilon_t \) is iid \( N (0,1) \) normally distributed random variable

The one step ahead forecast is simply the square root of the variance. The parameter \( q \) could be estimated by minimizing the error on a particular training set. The discussion of theoretical properties of the ARCH process could be found in (Tsay, 2005).

The name ARCH refers to this structure: the model is autoregressive, since \( \varepsilon_t \) clearly depends on previous \( \varepsilon_{t-1} \), and conditionally heteroskedasticity, since the conditional variance changes continually.

4.3 GARCH (p, q)

The model is very parsimonious and considered to be sufficient for capturing the volatility clustering without the necessity of using higher models GARCH (1, 1) model is as follows:
Where $h_t$ = conditional variance, $\varepsilon_t$ = residual at time $t$.

4.4 GARCH – M

This model allows the conditional mean to be the function of the conditional variance. The model is as follows:

$$y_t = \mu + \delta \sqrt{h_{t-1}} + \varepsilon_t, \varepsilon_t \sim N(0, \sigma^2_t)$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + \beta_1 h_{t-1}$$

Where $y_t$ = stock return and $\mu$ = mean. Rest all the coefficients must be non-negative to ensure the conditional variance.

$\delta$ = risk premium parameter. If its value is greater than zero + statistically significant, it shows that the returns are positively related to volatility.

Moreover, GARCH – M model states that serial correlations among the return series exist and can be exhibited by conditional variance equation.

4.5 TARCH (Threshold ARCH)

TARCH and EGARCH are the asymmetric models to model volatility. According to Ender, the threshold shocks that are above or below the threshold affect the volatility in their own different ways. The conditional variance equation is as follows:

$$h_t = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + \beta_1 h_{t-1} + \gamma \varepsilon^2_{t-1} I_{r-1}$$

The value of $\gamma$ should be strictly greater than zero so as to show the capability to take into consideration the leverage effect of the TARCH models. Threshold ARCH (TARCH) process Zakoïan (1994) and Glosten et al. (1993) applied the TARCH model with a purpose of independence than for the asymmetric effect of the “news”. The TARCH (p, q) specification is given by:

$$\sigma^2_t = \omega + \sum_{j=1}^{p} \beta_j \sigma^2_{t-j} + \sum_{j=1}^{q} \gamma_j u^2_{t-j} I_{r-1}$$

where,

$$I_{r-k} = \begin{cases} 
1 & \text{if } u_t < 0 \\
0 & \text{otherwise}
\end{cases}$$

In the TARCH model, “good news”, $u_t > 0$ and “bad news”, $u_t < 0$ have different effects on the conditional variance. When $\gamma_k \neq 0$, it can be concluded that the news impact is asymmetric and that there is presence of leverage effects. The difference between the TARCH and EGARCH
is that TARCH assumes leverage effect as quadratic and the EGARCH assumes leverage effect as exponential.

4.6 EGARCH

In this model there is no need of directly performing the conditional variance, instead, this model incorporates the logarithm of the conditional variance. The equation is as follows:

$$\ln h_i = \alpha_0 + \beta_1 \ln h_{i-1} + \left[ \alpha_1 \frac{|\epsilon_{i-1}|}{\sqrt{h_{i-1}}} + \gamma \frac{\epsilon_{i-1}}{\sqrt{h_{i-1}}} \right]$$

In this model the nonnegative constraint is not necessary because the conditional variance is already non-negative. Moreover, the parameter $\gamma$ generates the leverage effect as the yesterday’s shock is incorporated in the model.

The GARCH model is not the best model to explain the “leverage effects”, which are often observed in financial time series. Concept of leverage effects, which were first observed by Black (1976), is related to the fluctuation in the stock prices which seemed to be inversely related to the fluctuation in the stock volatility. One can deduce that the effects of a shock on the volatility are asymmetric or in other words, one can say that the effect of good news, a positive lagged residual, may be different from the effects of the bad ones, a negative lagged residual. Development and the presentation of EGARCH model by Nelson (1991) which accounts for an asymmetric response to a shock. A commonly used model is the EGARCH (1, 1) given by:

$$\log(\sigma_i^2) = \alpha_0 + \alpha_1 \frac{|\epsilon_{i-1}|}{\sigma_{i-1}} + \beta_1 \log(\sigma_{i-1}^2) + \gamma \frac{\epsilon_{i-1}}{\sigma_{i-1}}$$

The term $\gamma$, accounts for the presence of the leverage effects, which makes the model asymmetric. When the asymmetric model for volatility is applied, it allows the volatility to respond, more readily, when the prices are falling due to the bad news than with corresponding increases due to the good news.

5. Empirical results

Previous empirical studies have employed numerous error statistics to compare the forecast performance of GARCH type models. The most popular statistics used in those papers are root mean squared error (RMSE), mean absolute error (MAE) and mean absolute percentage error (MAPE). Following those papers and consistent with this research, the forecast errors generated
from the various models in this study are compared by those error statistics which are defined as follows:

\[
MB = \frac{1}{N_K} \sum_{t=1}^{N_K} (\hat{\sigma}_t^2 - \sigma_t^2)
\]

\[
MAE = \frac{1}{N_K} \sum_{t=1}^{N_K} |\hat{\sigma}_t^2 - \sigma_t^2|
\]

\[
RMSE = \sqrt{\frac{1}{N_K} \sum_{t=1}^{N_K} (\hat{\sigma}_t^2 - \sigma_t^2)^2}
\]

\[
MAPE = \frac{1}{N_K} \sum_{t=1}^{N_K} \left(\frac{|\hat{\sigma}_t^2 - \sigma_t^2|}{\sigma_t^2}\right)
\]

\[
U = \sqrt{\frac{1}{N_K} \sum_{t=1}^{N_K} (\hat{\sigma}_t^2 - \sigma_t^2)^2} + \sqrt{\frac{1}{N_K} \sum_{t=1}^{N_K} (\hat{\sigma}_t^2)^2}
\]

In the entire above statistics, ‘n’ stands for the number of out of sample forecasts. Theil’s U is a statistic that uses the random walk as a benchmark for comparing the forecasting ability of the models. Behind this notion is the belief that if a forecasting model cannot do better than a naïve forecast, then the model is not doing an adequate job. These statistics are generally termed as symmetric forecast error statistics as they penalize both for over forecast and for under forecast equally. Since over forecast and under forecast may lead to different profit/cost consequences to buyers and sellers differently they have to be treated differently i.e., a differential weighting is required since under forecast of volatility may be desirable for an option buyer since the resulting price will be less but it will be undesirable for the seller as the buyer’s gain will be the seller’s loss, following Brailsford and Faff (1996).

However, it should be noted that the evaluation of the forecast in fact varies a little and are dependent upon the choice of the error metric. Nonetheless, there is a reasonably high degree of consistency across the metrics that provide lower values for the second method across all currencies. On the benchmark basis, the model with the lowest relative statistic value is supposed to be the best forecasting model.
6. Results and discussion

The comparison of the various models is done on the basis of the two most used model selection criteria. They are Akaike Information Criteria (AIC) and Schwarz’s Bayesian Information Criteria (BIC / SIC).

Mallow’s Cp is (almost) a special case of Akaike Information Criteria:

\[ \text{AIC (M)} = -2 \log L(M) + 2 \cdot p(M) \]

Where \( L(M) \) is the likelihood function of the parameters in model M being evaluated at the MLE (Maximum Likelihood Estimators).

Schwarz’s Bayesian Information Criterion (BIC)

\[ \text{BIC (M)} = -2 \log L(M) + p(M) \cdot \log n \]

THUMB RULE: The smaller the value of AIC and BIC, the better the model. The better model suggested according to the AIC and SIC criterion is TARCH / EWMA and ARMA (1,1)/ARCH/EGARCH for stock and foreign exchange market respectively.

However, these criterions alone cannot be used to arrive at conclusion and the values forecast error statistics as discussed in earlier section are included to have near accurate decision.

Table 2 presents the error statistics results explained in the earlier section. Firstly, based on Theil’s-U and MAE the model that outperforms others is:

- GARCH (5,0) model for the NSE index return
- ARMA (1,1) and EWMA model for BSE index return
- EGARCH for exchange rate data.

While on the basis of MAPE:

- EGARCH and TARCH for stock market
- PARCH and EGARCH for foreign exchange market.

In the stock market EGARCH is found to perform better on the basis of three measures – RMSE and MAPE and Theil’s U, however, the difference in forecast error as per GARCH (5, 0) and EWMA is very nominal. On the other hand the TARCH models perform clearly ahead of EWMA in the forex market on the basis of three measures.

In the stock market the forecast accuracy increases on an average of 40% by using the TARCH, ARMA and EGARCH models.
7. Conclusion

A total of eight different models were considered in this study and these competing models were evaluated on the basis of two classes of evaluation measures – symmetric and asymmetric error statistics. Based on the out of sample forecasts and the number of evaluation measures that rank a particular method as superior we can infer that EWMA and TARCH will lead to improvements in volatility forecasts in the stock market and the EGARCH will achieve the same in the forex market. These findings are contrary to the findings of Brailsford and Faff (1996) who found that a single method is not superior. But the results in stock market are similar to the findings of Akigray (1989) and McMillan et al (2000) and Anderson and Bollerslev (1998) and Anderson et al (1999) in the forex market. The inferences remain same even on the basis of asymmetric error statistics. This would suggest the superiority of the entailed models over others given the stable future situations. Any crisis period or unstable situation might lead to variable results from the concluded models. Therefore readers are suggested to make changes accordingly.

Table: 1

<table>
<thead>
<tr>
<th>Statistics</th>
<th>NIFTY return</th>
<th>SENSEX return</th>
<th>Exchange rate return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.045888</td>
<td>0.068931</td>
<td>-0.007033</td>
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<tr>
<td>Median</td>
<td>0.060000</td>
<td>0.080000</td>
<td>0.000000</td>
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<tr>
<td>Maximum</td>
<td>17.74000</td>
<td>17.34000</td>
<td>3.755964</td>
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<tr>
<td>Minimum</td>
<td>-12.24000</td>
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<td>Standard deviation</td>
<td>1.611443</td>
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<td>Skewness</td>
<td>0.033508</td>
<td>0.087580</td>
<td>0.084445</td>
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<tr>
<td>Kurtosis</td>
<td>9.844356</td>
<td>8.806592</td>
<td>13.39618</td>
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<tr>
<td>Jarque-Bera</td>
<td>9237.200</td>
<td>7850.440</td>
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<tr>
<td>Probability</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
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Table: 2

1. For NSE return series

<table>
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<tr>
<th>Models</th>
<th>TU</th>
<th>RMSE</th>
<th>MAE</th>
<th>MAPE</th>
</tr>
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<tbody>
<tr>
<td>Historical volatility models</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>EWMA</td>
<td>0.952</td>
<td>1.70</td>
<td>1.17</td>
<td>106.92</td>
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<td>Autoregressive an Heteroskedastic models</td>
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<tr>
<td>ARMA(1,1)</td>
<td>0.922</td>
<td>1.69</td>
<td>1.17</td>
<td>1.13</td>
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2. For BSE index

<table>
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<th>MAPE</th>
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<td>Historical volatility models</td>
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</tr>
<tr>
<td>EWMA</td>
<td>0.94</td>
<td>1.624</td>
<td>1.121</td>
<td>1.08</td>
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<td>Autoregressive an Heteroskedasticity models</td>
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<tr>
<td>ARMA(1,1)</td>
<td>0.911</td>
<td>1.625</td>
<td>1.122</td>
<td>1.10</td>
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<tr>
<td>ARCH(5)</td>
<td>0.94</td>
<td>1.627</td>
<td>1.120</td>
<td>1.13</td>
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<tr>
<td>GARCH (1.1)</td>
<td>0.942</td>
<td>1.627</td>
<td>1.120</td>
<td>1.13</td>
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<tr>
<td>EGARCH</td>
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<td>1.627</td>
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<td>1.08</td>
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<td>GARCH(2,2)</td>
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<td>PARCH</td>
<td>0.954</td>
<td>1.627</td>
<td>1.120</td>
<td>1.09</td>
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3. For exchange rate data

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<tr>
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<th>RMSE</th>
<th>MAE</th>
<th>MAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Historical volatility models</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EWMA</td>
<td>0.972</td>
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<td>.00399</td>
<td>0.9754</td>
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<tr>
<td>ARMA(1,1)</td>
<td>0.924</td>
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<tr>
<td>ARCH(5)</td>
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<td>.0056</td>
<td>.00399</td>
<td>0.975</td>
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<tr>
<td>GARCH (1.1)</td>
<td>0.995</td>
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**Figure 1(a)**

**Figure 1(b)**
References


Gavrishchaka V.V, Ganguli S.B, Optimization of the neutral-network geomagnetic model for forecasting large-amplitude substorm events, J. Geophys.


Schittenkopf C, Doroner G, Dockner EJ, Volatility prediction with mixture density network

