A New Approach to Asset Pricing

Abstract

This paper presents a new approach to portfolio management by providing preliminary empirical evidence that asset prices are linear polynomials of multiple basic factors - asset volumes, index price, index volume, time and preceding asset prices. Hence asset returns are nonlinear rational functions that do not add linearly in a portfolio, especially when they are averaged out over multiple time intervals. However, for time series data observed on single time interval basis, returns may be treated as approximately linear and modeled directly through a multiple regression model using the basic variables mentioned above and other relevant market factors. Accordingly, two new asset pricing models have been developed for average returns and continuous returns respectively that provide more accurate estimates as compared to existing linear models.

Keywords: Returns, Volumes, RF Model, CAPM, Fama French Model,

JEL Codes: G11, G12

A New Approach to Asset Pricing

The theory of modern portfolio management and asset pricing rests on the foundations laid by Markowitz (1952). He defined risk mathematically for the first time being the variance in returns. He also further propounded the twin objectives of maximizing returns while minimizing risks as the goal of all rational management of such investments and as a result the model of portfolio selection is also called as a 'Mean Variance Model'. However, the Markowitzian framework of portfolio management assumes that all stock returns add algebraically in a portfolio and based on this assumption the Capital Asset Pricing Model (CAPM), given by Sharpe (1964) and Lintner (1965), advocates a linear relationship between the asset returns and the returns of the market proxy index. But, though the CAPM led to the very important revelation of the connection between the asset returns and the market returns, it did not quite match the actual empirical evidence. Many empirical studies of the model reported that the actual returns were consistently and systematically found to plot out much flatter than the estimated CAPM returns across increasing market risk. Hence, though the actual asset returns were found to be positively related to the market returns, as stated by the CAPM, they were found to be much slower in increasing with increasing risk resulting in a flatter, nonlinear plot. This mismatch of the CAPM estimates with the actual empirical returns could be due to the oversimplified assumptions used in developing the model or due to market inefficiencies. The two main assumptions used in the CAPM are: a) 'complete agreement' among the investors regarding the joint distribution of the asset returns during a given time interval of t-1 to t and b) a risk free lending rate. Due to the empirical mismatch of the CAPM, a few other linear asset pricing models were put forth in order to improve upon the CAPM estimates. Among these, the Fama-French-3-Factor model (FF3F) developed by Fama and French (1996, 2004) is the most noteworthy and widely used today. However, even the FF3F is a linear model that uses two additional factors in addition to the market returns based on the a) Size of the firm and b) Value of the Stock. Thus, even though these two additional factors improved upon the returns estimates, the theoretical motivations behind including these factors remained unexplained. Further, though the FF3F improved the accuracy of the estimated continuous returns, being a linear model it is again not able to fit the nonlinear empirical average returns. So the theoretical reasons behind the discrepancies between the CAPM estimates and the actual returns stay unexplained. This gives rise to a research gap that needs to be addressed especially since today's finance world has become more sophisticated and demanding due to ongoing technological progress. Consequently, it is of the utmost importance that the theory of the asset markets should match the empirical reality.

Accordingly, this paper has attempted to fill the above mentioned research gap by putting forth a new theory that attributes the above mentioned discrepancies between the theoretical and the empirical asset returns as being due to the non-linearity of the asset returns and the interplay of other deciding factors. Asset returns behave nonlinearly, especially when they are averaged over multiple time intervals since they are rational functions of their prices. This paper provides preliminary empirical evidence from the US, Australian and Indian markets, that the stock returns are rational functions of index price, index volume, preceding stock prices, stock volumes and time trends, thus behaving non-linearly across risk, especially when averaged out across time. The continuous returns behave approximately linearly but again their estimation can be further improved upon by including additional relevant factors mentioned above.

2. Brief Literature Review

As already mentioned above, the CAPM is the most simple and still a highly popular asset pricing model in spite of its empirical drawbacks. It was given by Sharpe (1964) and Lintner (1965) as follows:

$$E(R_{i,t}) = R_{f,t} + \beta_{i,m}\,[E(R_{m,t}) - R_{f,t}],\ i = 1, 2,.,N. \qquad \ldots(1)$$

Here, $R_{f,t}$ is the risk free rate of return at time 't', $E(R_{i,t})$ and $E(R_{m,t})$ are the expected asset and the expected market returns respectively while $\beta_{i,m}$ is the market risk of the asset 'i'. However, despite its simple intuitive appeal, the CAPM was not able to accurately estimate the actual returns. Numerous subsequent empirical studies (Douglas 1968; Friend and Blume 1970; Miller and Scholes 1972; Blume and Friend 1973; Fama and MacBeth 1973; Stambaugh 1982; Fama and French 1992; Fama and French 2004 etc.) reported differences in the theoretical asset returns predicted by the CAPM and the empirical returns that were actually observed. They found that across increasing market risk $\beta_{i,m}$, the actual asset returns $R_{i,t}$ plotted out much flatter than the ones estimated through CAPM. Thus, the actual asset returns were found to be higher than the CAPM returns for lower risk and lower for higher risk. So, even though the actual asset returns were found to have a general positive relationship with the market premium $(R_{m,t} - R_f)$, their plots were found to be nonlinear and much flatter than that of the CAPM estimates. The findings of these studies were summarized by Fama and French (2004) who themselves provided a very comprehensive empirical evidence based on the monthly returns of US stocks from 1928 to 2003 as roughly represented in Figure 1. The Figure 1 shown below is an approximate copy of the actual plot given by Fama and French (2004) whereby the average returns of the portfolios are plotted against their post-ranking betas that are obtained by regressing the post-ranking portfolio returns on the market returns. The solid line is the linear plot of the CAPM average returns while the dots represent the actual average returns.

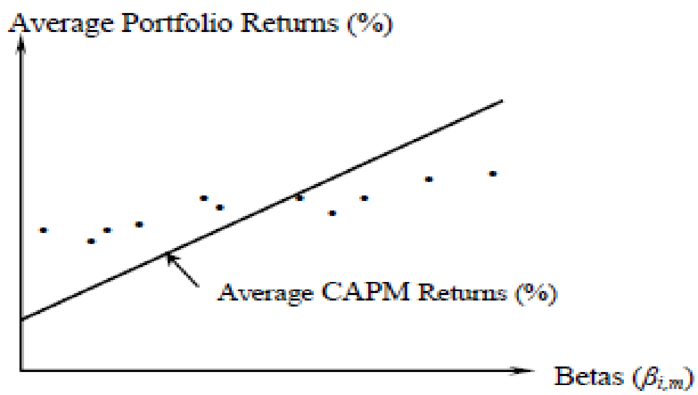

Figure 1: Plot of Actual vs. CAPM returns across $\beta_{i,m}$ (Fama & French, 2004).

Based on the above differences between the CAPM returns and the actual returns, Fama (Fama 1970, Fama and French 1996, 2004) put forward the Joint Hypothesis Problem (JHP) that these differences are due to either or both of the two possibilities – i) a flawed and incomplete asset pricing model that does not fully capture the empirical reality and/or ii) an inefficient market that does not abide by the idealistic assumptions of the theoretical model. In the meanwhile, various studies reported empirical evidence that various firm specific financial data like the size, price to earnings ratios etc. contained additional information about asset returns that are not fully reflected by the CAPM estimates (Basu 1977, Stattman 1980, Banz 1981, Bhandari 1988 etc.). Fama and French themselves developed a three factor model (Fama and French, 1993, 1996, 2004) that utilizes two additional measures besides the index return, viz. the size of the firm and the value of the stock as measured by the book to market equity (B/M) ratio. The Fama French three factor model (FF3F) was found to capture variations in the expected stock returns that are not explained by the CAPM. The FF3F is given by:

$$E(R_{i,t}) - R_{f,t} = \beta_{i,m} [E(R_{m,t}) - R_{f,t}) + \beta_{i,s} E(SMB_t) + \beta_{i,h} E(HML_t) \qquad \ldots(2)$$

Here, SMB_t (small minus big) is the difference between the returns of the diversified portfolios of small and big stocks sorted on the basis of their sizes, i.e. their market capitalizations. Similarly, HML_t (high minus low) is again the difference between the returns of the diversified portfolios of high and low B/M stocks. The FF3F is now generally chosen over the CAPM because of its greater accuracy. However, according to Fama and French themselves, the FF3F suffers from the limitation that its two additional factors were included through empirical motivations while their underlying rationales remain unrevealed. Besides this, the second important drawback of the FF3F is that it is a linear regression model for the average returns whereas the empirical evidence indicates a nonlinear plot as evident from the Figure 1. This signifies that even though FF3F had improved upon the CAPM estimates, there is still further scope of improvement of its accuracy.

3. New Asset Pricing Theory and Models

To resolve the above research gap, a new asset pricing theory and model has been developed in this paper wherein it has been explained that the above discussed discrepancies between the theoretical and the actual returns are due to the fact that asset returns do not add linearly in a portfolio. This is because the asset prices themselves are polynomials deriving from various relevant firm-related and market-related factors and the asset returns being ratios of consecutive prices are 'rational functions'. Based on this, the new theory is called the 'Rational Function' (RF) theory. The RF theory has been built by re-visiting the basic fundamentals of Supply and Demand functions that govern the pricing of an asset through the quantity being traded. Many papers on technical analysis, classical academics and Causality Studies (Murray 1994, Polonchek and Krehbiel 1994, Pisedtasalasai and Gunasekarage, 2006) have included Volume and time trends in determining asset returns but these papers have included these factors as motivated through patterns and not through any theoretical reasoning. The supply-demand framework provides this missing economical reasoning. Here, the quantity of the stock being traded is also called as the 'Volumes' in the language of the capital markets. In general, the supply curve represents the quantities of an asset that the suppliers or sellers are willing to sell at different prices while the demand curve represents the quantities of the asset that the consumers or buyers are willing to buy at different prices, assuming *ceteris paribus*, i.e. all other factors remaining unchanged. The Supply-& Demand model was developed and popularized by Alfred Marshall in his 1890 textbook "Principles of Economics", whereby the price is plotted along the vertical axis and the quantity along the horizontal axis. Thus, supply and demand functions together form the basis of price setting in a free market environment, such that the unit price for the traded good or asset varies freely till it settles at a point where the quantity demanded by the buyers will equal the quantity supplied by the sellers, resulting in an economic equilibrium of price and quantity. The supply and demand functions are basically rooted in the rational market theory of profit maximization, whereby the buyers are expected to buy more at lower prices while the sellers are expected to sell more at higher prices. Since the buyers tend to buy more at lower prices, the demand curve would have a negative slope while the supply curve would have a positive slope as sellers tend to sell more at higher prices. These curves could be linear, but following the laws of increasing marginal cost for supply curve and decreasing marginal utility for demand curve, we tend to get exponential and logarithmic curves respectively. It may be noted that this framework of price-determination, through supply and demand forces, obviates the necessity of the unrealistic assumption of 'complete agreement' among the investors about the expected return of an asset, used in linear asset pricing models like CAPM.

Further, when working directly with the prices, the assumption of a risk free rate of lending and borrowing that was used for CAPM and FF3F has also been dropped. For modelling the asset prices, the intercept is assumed to be zero because that would be the lowest price that could be payable for an asset. The intercept cannot be negative as no asset has negative price in a supply-demand framework but at the worst, an asset may lose all its value in a free market environment.

Thus, an asset price is a function of its own preceding price, its own volume of trade, the market return (as represented by a suitable index proxy), the change in market volume and the time trend and cyclical patterns inherent in its series. The market return is found to directly influence the asset price formation. The time trend component of the asset price is represented by the exponential variation of the asset value with respect to the simple chronological rank of the day '*t*' in the whole sample of the time series of the prices being analyzed. In the practical scenario, the change in asset volume is overshadowed by the change in volume of the market index and hence only the latter has been considered in the RF theory.

Accordingly, the RF Model (RFM) for asset pricing was derived as follows:

$$p_{i,t} = \beta_{i1}[\{(1 + ln(p_{m,t}/ p_{m,t-1})\ p_{i,t-1}\}] + \beta_{i2}[ln(v_{m,t}/v_{m,t-1})]^2 + \beta_{i3}(-0.1)^{t_t} \ldots(3)$$

Here, $p_{i,t}$ and $p_{i,t-1}$ are the asset prices on days '*t*' and '*t-1*' respectively while $p_{m,t}$ and $p_{m,t-1}$ are the index prices. Similarly, $v_{m,t}$ and $v_{m,t-1}$ are the corresponding index volumes and t_t is the time factor based on the chronological rank of the day '*t*' in the whole sample. It may be noted that the role of the market proxy in the above model is in shaping the price formation of the asset as a portion of the current price $p_{i,t}$ varies in direct proportion with the market return as $\{1 + ln(p_{m,t}/ p_{m,t-1})\ p_{i,t-1}\}$.

The above RFM equation is used for estimating the 'average' asset returns whereby the time series of prices are computed and the average returns are then obtained as the ratios of the average prices on two consecutive days '*t-1*' and '*t*'. However, when studying the time series of 'continuous' asset returns, it was found that these returns behave approximately linearly and hence should be modeled directly from the relevant firm specific and market specific factors. This may be compared to the scenario whereby the Earth though spherical in shape as visible from space is treated as flat for trigonometrical calculations done for land surveys. For this, the FF3F was found to provide quite accurate estimates through the relevant market-specific terms $R_{f,t}$ and $R_{m,t}$ and the firm-specific terms SMB_t and HML_t. Nonetheless, the accuracy of the FF3F estimates for the 'continuous' asset returns could be further improved by including three more factors derived from the RF theory, which are the market volume, time factor and the preceding asset returns. It may be noted that the time factor for modeling the continuous returns is different from that used for modeling prices. This is because that while asset prices are influenced by the exponential variation of the time-ranks, the continuous returns are shaped by the quadratic variation of the time-ranks. Thus, for studying the 'continuous' asset returns, a model was derived by combining the FF3F model with the RF theory to give the following combined RFM&FF3F equation:

$$R_{i,t}-R_{f,t}= \beta_{i,m}(R_{m,t}-R_{f,t})+\beta_{i,s}SMB_t+\beta_{i,h}HML_t+\beta_{i,v}[ln(v_{m,t}/v_{m,t-1})]+\beta_{i,o}(t_t)^2+\beta_{i,l}(R_{i,t-1}) \quad ..(4)$$

It should be clarified here itself that, $R_{i,t}$=ln($p_{i,t}/p_{i,t-1}$); $R_{m,t}$=ln($p_{m,t}/p_{m,t-1}$); and $V_{m,t}$=ln($v_{m,t}/v_{m,t-1}$). Thus, in this paper, the asset returns have been studied in two different formats – a) Average returns and b) Continuous returns.

a) Average Returns – This format considers assets that have been ranked according to their returns variances over the last 12 observations. The time series of prices of these ranked assets are then computed as per RFM equation (3) and then averaged across the cross section of a portfolio to give the price series of the diversified portfolios. These portfolio prices are then again averaged from 't' to 't+n-1' intervals and from 't+1' to 't+n' intervals, to obtain the average returns from the ratio of these average prices. These average portfolio returns are then listed according to their increasing risk and studied. This format is useful for plotting the risk-return profile of the assets and thus choosing the most mean-variance efficient asset portfolios.

b) Continuous Returns – This format studies continuous asset returns across both increasing risk and time using the same time series of portfolio prices ranked according to their riskiness, used for the earlier format of average returns discussed above. However, here the average returns are not computed from the time series of the asset prices. Instead, we compute the time series of continuous asset returns from the asset prices. This format is useful in describing the contemporaneous asset returns across increasing risk and time. As already mentioned, the continuous portfolio returns were found to behave 'approximately' linearly since this format uses data on a single time interval basis. Hence, for this format the asset returns were modeled directly as per the RFM&FF3F equation (4).

4. Methodology for the Empirical Tests

The main objective of this paper is to provide preliminary empirical evidence on the RFM concepts. Accordingly, sample portfolios have been collected from three different markets– USA, Australia and India covering different time windows in order to demonstrate the free empirical validity of the RF theory. The USA samples S1 to S8 have been constructed out of the constituent stocks of three indices of various sizes - the 30 stocks of the Dow Jones Industrial Average (DJIA) as on April 30, 2013, the 395 stocks constituting the Barron's 400 (B400) index as on August 01, 2013 and the 500 stocks constituting the S&P 500 index as on August 01, 2013. The monthly returns of the DJIA stocks were studied over the last ten years while the daily returns of the various US stocks have been considered from December 12, 2012 to April 30, 2013. The difference in the dates of selection of the components of DJIA and B400 and S&P500 is in order to add variety and depth to the analysis and to demonstrate the robustness of the RFM to the choice of the stocks and time-periods. As both small and large sample analyses of stocks are needed to test out any new investment theory, this paper has attempted both by considering sample sizes as small as 30 and as large as 500. The B400 was considered not only because of its large size of components but also because its components are chosen on the basis of their superior performance which is different from the selection criteria of DJIA and S&P500 that consist of companies with large market caps. Similarly two sample portfolios were constructed of the 100 pooled components of the S&P ASX 50 and S&P ASX Mid-cap 50 indices. The Australian samples were used to study their daily returns for 95 and 120 day time-windows contained within April 12, 2013 to September 30, 2013. Finally a sample of 30 stocks of the BSE Sensex as on January 1, 2005 was collected from India and their monthly returns were studied for an eight-year period from January 2002 to November 2009. Thus overall, the samples selected for this study represent a diversified view of the US, Australian and Indian markets from various time windows and the findings may be taken to be free from any selection biases. The daily price and volume data for the stocks as well as the indices for the USA and Australian markets were collected from the databases purchased from a licensed vendor called Norgate Investor Services which is based in Australia. The Indian data was collected from the Prowess Database provided by Centre for Monitoring Indian Economy (CMIE). The details of the various samples S1 to S11 that have been collected and studied have been provided in Table 1. Besides the stock market data, the monthly and the daily time series data for the $R_{f,t}$ values and the SMB_t and the HML_t factors used in the FF3F model for the US and the Indian markets, were collected from Kenneth R. French's website, wherein the data for the US and the Asia Pacific regions have been provided. For the Australian market the $R_{f,t}$ was dropped as it was mentioned on Kenneth French's website that the daily values of $R_{f,t}$ were negligible for the time period being studied. The Australian FF3F factors

SMB_t and HML_t were obtained by computing the difference in portfolio returns formed by sorting the 100 companies used for this study on the basis of a) their market capitalizations for SMB_t and b) their B/M ratios for HML_t over the two quarters during April 2013 to September 2013 using the archived accounting data for these companies provided by the Australian Financial Review. The collected data were then arranged into separate samples S1 to S11 as shown in Table 1.

The stocks in each of these samples were first sorted as per increasing risk as measured by the variances of their returns through a rolling time frame of the last 12 observations and then regrouped into five smaller portfolios P1 to P5 consisting of 6 stocks each for DJIA, 79 stocks each for B400, 100 stocks each for S&P500, 20 stocks each for the Australian samples and 6 stocks each for BSE Sensex, wherein P1 consists of the stocks of lowest variances while P5 contains the stocks of the highest variances. Besides these 5 portfolios, the returns of the full sample portfolio (P-full) consisting of all the sample stocks were also analyzed and studied. After sorting the stocks, the ranked series of stock prices were reconstructed from the actual stock returns for each rank, using some common base number (like 100), so as to avoid the sudden abrupt changes in the prices of these rank-stocks after each sorting. The above data for the samples S1 to S11 were then analyzed using the CAPM, the FF3F model and the RFM equations (3) and (4) for the two different formats, based on 'average' and 'continuous' returns respectively, as discussed above. These results have been reported in Tables 2 to 10. The estimated returns obtained for the both the formats by the CAPM, the FF3F and the RFM models have been compared for their empirical validity through their correlations with the actual returns as well as through their Sum of Squared Errors (SSE) with respect to the actual returns for each of the six portfolios P1 to P-full within each sample S1 to S11.

In addition to the Tables 2 to 10, the results of the analyses using equation (3) for the 'average' returns have been plotted as eleven charts for the samples S1 to S11 that have been given under Figure 2. The charts provided under Figure 2 pictorially indicate the empirical veracity of the estimated returns and can be used to form expectations of the risk-return profiles of the various assets that are being studied.

5. Results and Discussion

As already mentioned, all the results have been reported in the Tables 2 to 10. To start with, it is obvious from the Tables 2, 3, 4 and 5 that the t-statistics of the slopes for the index price is the highest for all the models, thus indicating that it is the most important factor in all the equations. As given in Table 2, the t-statistics of the $\beta_{i,m}$ for the CAPM are in the range of 18.35 to 74.68. The t-statistics for $\beta_{i,m}$ in the FF3F model are in the range of 17.74 to 77.31 as listed in Table 3. From the Table 3, it can be further seen that for the FF3F model, the t-statistics of $\beta_{i,s}$ and $\beta_{i,h}$ are much lower than those for $\beta_{i,m}$ and are in the range of -1.36 to 8.71 and -2.02 to 2.08 respectively. This indicates that the market return $R_{m,t}$ is the most important variable in determining the asset returns. Similarly, the t-statistics of the slopes for the market return factor $\beta_{i,1}$ for RFM equation (3) as shown in Table 4, are the highest and are in the range of 274.32 to 3463.38.

However, it can be seen from the results given in Table 6 that the 'average' returns estimated by the RFM equation (3) have consistently the highest correlations with the actual average returns and are all above 98%. On the other hand, the correlations of the CAPM and the FF3F estimates are quite unpredictable and are both negative and positive in values. The t-statistics of the correlations are again highest for the RFM estimates and lie in the range of 9.09 to 127.87. This shows that the 'average' returns computed from the RFM equation (3) are the most accurate. This conclusion is further strengthened by the results given in Table 7 and the charts provided under Figure 2. Further, on comparing the correlations between the CAPM and the FF3F estimates with the actual average returns, it is seen that both CAPM and FF3F provide nearly similar results and FF3F provides no clear advantage over the CAPM. It is only the RFM estimates that are clearly more accurate than both CAPM and RFM estimates. In fact, the negative correlations of the CAPM and the FF3F estimates with the actual 'average' returns show that these linear models do not depict the true non-linear picture of average asset returns across increasing risk as obvious from the charts in Figure 2, even though they might have identified important and relevant factors influencing the asset returns. This proves that the asset returns are basically non-linear in nature as they are the rational functions of prices. As the charts of Figure 2 show that the RFM estimates of the 'average' returns are nearly same as those of the actual returns while that is not so for the other two linear models CAPM and FF3F, it proves the basic empirical validity of the RFM theory.

The same conclusions are borne out by the values of the Sum of Squared Errors of Average Returns (SSEA) as computed from the difference in values of the estimates and actuals, which are given in Table 7, indicating that the RFM estimates for average returns obtained from the equation (3) are consistently most accurate. The RFM estimates for the 'average' returns provides improvements of over 90% to the CAPM and FF3F estimates. Thus, it follows that though the index price might be the most important variable in computing asset returns, by itself it cannot provide the important improvements brought in by the nonlinearity and other factors used in RFM.

However, for the 'continuous' returns, the asset returns behave approximately linearly and should be modeled directly through linear regression models as already mentioned in the preceding sections. Thus for studying the continuous returns, the FF3F model has been combined with three additional factors identified by the RF theory, which are market volumes, time and preceding returns as shown in the combined RFM&FF3F model equation (4). As can be seen from the Table 8, the average correlations between the actual 'continuous' returns and the RFM estimates are again marginally, but consistently, higher than those of the CAPM and the FF3F estimates. This shows that the RFM definitely provides a chance to improve the FF3F estimates for the continuous returns as well. Even though the FF3F data is supposed to work best for large samples having been specifically obtained from them, the RFM&FF3F model provides better estimates.

Similarly, it can be seen from the Table 9, that the Sum of Squared Errors (SSE) between the actual 'continuous' returns and the RFM&FF3F 'continuous' returns are all less than those of the CAPM and the FF3F estimates. It is also evident that the FF3F estimates are all more accurate than the CAPM estimates but the combined RFM&FF3F estimates have the least errors and hence are the best. The improvements in SSE provided by the combined RFM&FF3F model over the FF3F model are in the range of 1.27% to 8.28%. The improvements over the CAPM estimates lie in the range of 11.59% to 55.99%. The reason behind the generally higher accuracy of the FF3F estimates for S7 maybe because of the fact that the large number of the constituent stocks of B400 are chosen on the same performance principles as the ones that underline the construction of the factors SMB and HML in the FF3F model. However, it should be noted that finally, the combined RFM&FF3F model outperforms all the other models for the continuous contemporaneous returns. It can be seen from the Table 9 that the percentage improvements in errors are higher for the US markets and larger samples S7 and S8 at above 8%.

Further, it can be seen from the two bottom rows of Table 9, that a paired t-test for no improvement in the SSE values for the FF3F model over the CAPM model i.e. H_0: $(SSE_{CAPM} - SSE_{FF3F}) / SSE_{CAPM} \leq 0$ can be safely rejected at a p value of 1.39%. Similarly, paired t-tests for no improvement due to RFM&FF3F combined equation (4) over CAPM and FF3F equation can be rejected safely at pvalues of 0.67% and 0.14% respectively. This shows that the combined equation (4) provides definite improvements in accuracy over the CAPM and the FF3F models and has the least errors in its estimates. Thus, the RFM&FF3F equation (4) is a clear improvement over the CAPM and the FF3F models in estimating and describing the 'continuous' asset returns. Hence, even though the 'continuous' asset returns behave approximately linearly across increasing risk for time series analyses, yet the additional independent variables like market volumes, time factor and preceding returns that have been included apart from the index return, in the combined RFM are important in improving the accuracy of the estimated asset returns for this format.

6. Conclusions

The RFM estimates of the ‘average’ asset returns are more accurate than both the CAPM and the FF3F estimates as indicated by the results from the equations (3) that are tabulated in Tables 6 and 7 and plotted as charts under Figure 2. This shows that the average asset returns are non-linear in nature and hence do not add linearly in a portfolio. However, for practical purposes the ‘continuous’ asset returns should be treated as ‘approximately’ linear for time series data involving single time intervals and the returns should be modeled directly through linear regression techniques. Even then, the additional factors included in the combined RFM&FF3F equation (4) apart from the index return, like the index volume, the time trend and the preceding asset returns should be used for estimating the continuous asset returns for both cross-sectional as well as time-series data. The FF3F factors of Size and B/M ratio are found to increase the accuracy of the estimates of the ‘continuous’ returns over those of the CAPM model and hence should be used in combination with the RFM model. Since, both prices and volumes are important in RFM, this theory indicates that the asset price and the asset volume are complementary market forces, as the third basic factor – time is an uncontrollable passive factor. Further, the charts of the Risk Format indicate that risk-return-efficient investments should be carefully selected from such charts as sometimes the lower risk assets offer the higher returns. Thus, it seems that the RFM theory, if used judiciously, could help the investors to make better and more mean-variance efficient investments in the stocks as compared to the existing asset pricing models.

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Table 1: Names of Portfolios

S. No.	Portfolios	Market	Data from	to	Type of Returns	Number of Observations	Market Proxy	Names
1	30 components of DJIA as on April 30, 2013	USA	30-May-03	30-Apr-13	Monthly	120	DJIA	S1
2	30 components of DJIA as on April 30, 2013	USA	30-May-03	30-Apr-13	Monthly	120	S&P 500	S2
3	30 components of DJIA as on April 30, 2013	USA	30-Jun-05	30-Apr-13	Monthly	95	DJIA	S3
4	30 components of DJIA as on April 30, 2013	USA	30-Jun-05	30-Apr-13	Monthly	95	S&P 500	S4
5	30 components of DJIA as on April 30, 2013	USA	12-Dec-12	30-Apr-13	Daily	95	DJIA	S5
6	30 components of DJIA as on April 30, 2013	USA	12-Dec-12	30-Apr-13	Daily	95	S&P 500	S6
7	396 components of B400 as on August 1, 2013	USA	12-Dec-12	30-Apr-13	Daily	95	S&P 500	S7
8	500 components of S&P 500 as on August 1, 2013	USA	12-Dec-12	30-Apr-13	Daily	95	S&P 500	S8
9	100 components of S&P ASX 50 and S&P ASX Mid-Cap 50 as on May 15, 2013	Australia	20-May-13	30-Sep-13	Daily	95	ASX All Ordinaries	S9
10	100 components of S&P ASX 50 and S&P ASX Mid-Cap 50 as on May 15, 2013	Australia	12-Apr-13	30-Sep-13	Daily	120	ASX All Ordinaries	S10
11	30 components of BSE Sensex as on January 1, 2005	India	31-Jan-02	30-Nov-09	Monthly	95	BSE Sensex	S11

Table 2: Average t-statistics for slopes of CAPM: $R_{i,t} = R_{f,t} + \beta_{i,m} (R_{m,t} - R_{f,t})$

Sample Portfolios	Average t-statistics across P1 to P-full
	$\beta_{i,m}$
S1	25.66
S2	23.57
S3	25.00
S4	21.59
S5	20.06
S6	18.75
S7	24.11
S8	34.45
S9	23.25
S10	23.63
S11	74.68

Table 3: Average t-statistics for slopes of FF3F: $R_{i,t} - R_{f,t} = \beta_{i,m} (R_{m,t} - R_{f,t}) + \beta_{i,s} SMB_t + \beta_{i,h} HML_t$

Sample Portfolios	Average t-statistics across P1 to P-full		
	$\beta_{i,m}$	$\beta_{i,s}$	$\beta_{i,h}$
S1	23.32	1.11	0.98
S2	21.07	-0.91	1.53
S3	23.06	0.84	1.46
S4	19.68	-1.12	2.08
S5	19.12	0.05	1.08
S6	17.74	-1.36	0.36
S7	28.76	8.71	-2.02
S8	34.50	4.06	1.31
S9	21.85	2.60	-0.07
S10	22.40	4.01	0.69
S11	77.31	1.20	0.12

Table 4: Average t-statistics for slopes of RFM Equation:

$$p_{i,t} = \beta_{i1}[\{(1 + \ln(p_{m,t}/ p_{m,t-1})\, p_{i,t-1}\}] + \beta_{i2}[\ln(v_{m,t}/v_{m,t-1})]^2 + \beta_{i3}(-0.1)^t{}_t$$

Sample Portfolios	Average t-statistics across P1 to P-full		
	β_{i1}	β_{i2}	β_{i3}
S1	513.83	0.28	-0.98
S2	477.16	0.63	-0.10
S3	459.47	0.56	-0.11
S4	398.26	0.77	-0.02
S5	2632.44	-0.01	-0.27
S6	2303.79	0.98	1.00
S7	2511.50	-1.52	-0.85
S8	3463.38	-0.96	-0.41
S9	2355.99	-0.14	-0.51
S10	2485.67	-0.89	0.32
S11	274.32	-0.64	-0.02

Table 5: Average t-statistics for slopes of Combined RFM-&-FF3F Equation:

$$R_{i,t} - R_{f,t} = \beta_{i,m} (R_{m,t} - R_{f,t}) + \beta_{i,s} SMB_t + \beta_{i,h} HML_t + \beta_{i,v}[ln(v_{m,t}/v_{m,t-1})] + \beta_{i,o}(t_t)^2 + \beta_{i,l}(R_{i,t-1})$$

Sample Portfolios	Average t-statistics across P1 to P-full					
	$\beta_{i,m}$	$\beta_{i,s}$	$\beta_{i,h}$	$\beta_{i,v}$	$\beta_{i,o}$	$\beta_{i,l}$
S1	21.80	1.24	0.90	-0.92	0.06	0.24
S2	20.13	-0.90	1.66	-0.42	0.40	-0.73
S3	21.55	0.92	1.33	-0.72	0.16	0.04
S4	18.90	-1.13	2.17	-0.19	0.40	-0.77
S5	18.65	0.17	1.12	-0.10	0.64	-0.89
S6	17.08	-1.22	0.36	0.14	0.66	-0.21
S7	28.01	8.72	-1.89	-1.46	0.11	0.42
S8	33.55	4.09	1.42	-1.00	0.37	0.35
S9	21.14	2.67	-0.12	0.83	0.53	-1.20
S10	21.97	3.63	0.52	0.70	0.37	-0.52
S11	66.04	1.24	0.30	-0.33	0.43	-0.87

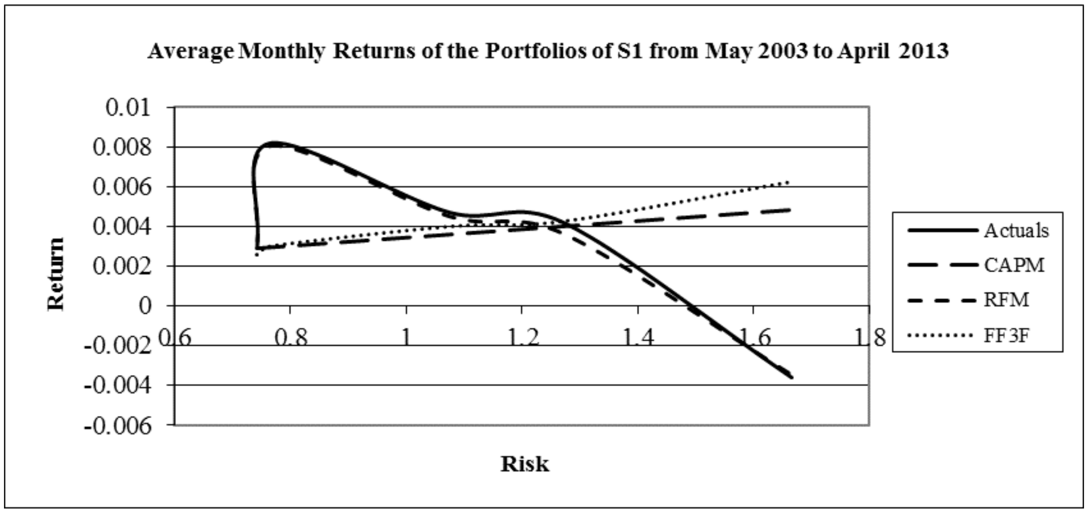
Table 6: Correlation of Estimated Average Returns with Actual Average Returns across P1 to P-full

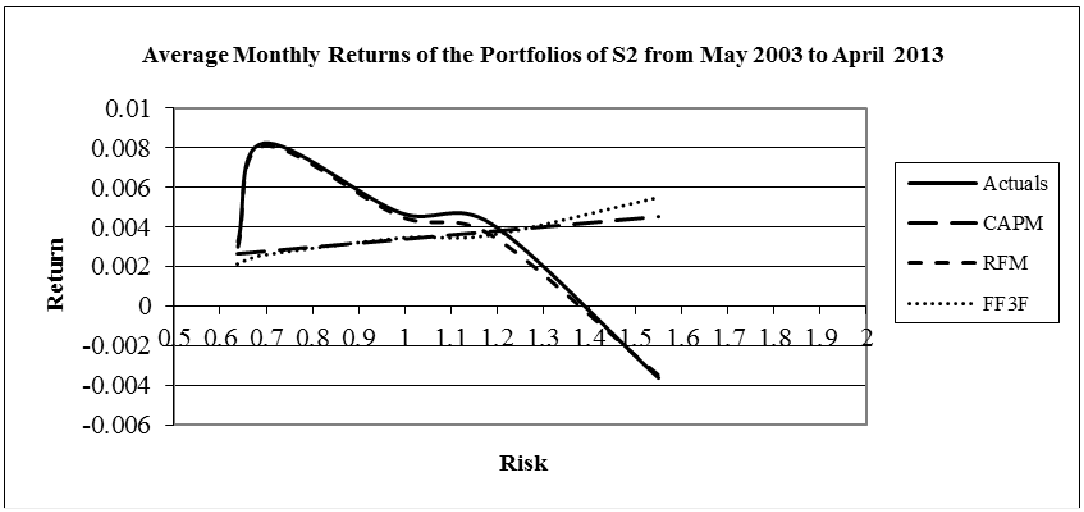
Portfolios	CAPM		FF3F		RFM	
	Correlation	t-stats	Correlation	t-stats	Correlation	t-stats
S1	-81.74%	-2.46	-81.75%	-2.46	99.88%	34.63
S2	-79.46%	-2.27	-81.53%	-2.44	99.87%	33.59
S3	-79.37%	-2.26	-67.55%	-1.59	99.75%	24.46
S4	-77.19%	-2.10	-58.10%	-1.24	99.73%	23.68
S5	46.82%	0.92	43.55%	0.84	98.23%	9.09
S6	52.44%	1.07	48.00%	0.95	98.77%	10.94
S7	33.23%	0.61	31.64%	0.58	98.27%	9.20
S8	-73.59%	-1.88	-63.86%	-1.44	98.65%	10.45
S9	25.07%	0.45	25.78%	0.46	99.86%	32.46
S10	65.35%	1.50	75.89%	2.02	99.95%	53.82
S11	39.22%	0.74	17.13%	0.30	99.99%	127.87

Table 7: Sum of Squared Errors of Average Returns (SSEA) of Estimated Average Returns as compared to Actual Average Returns across P1 to P-full

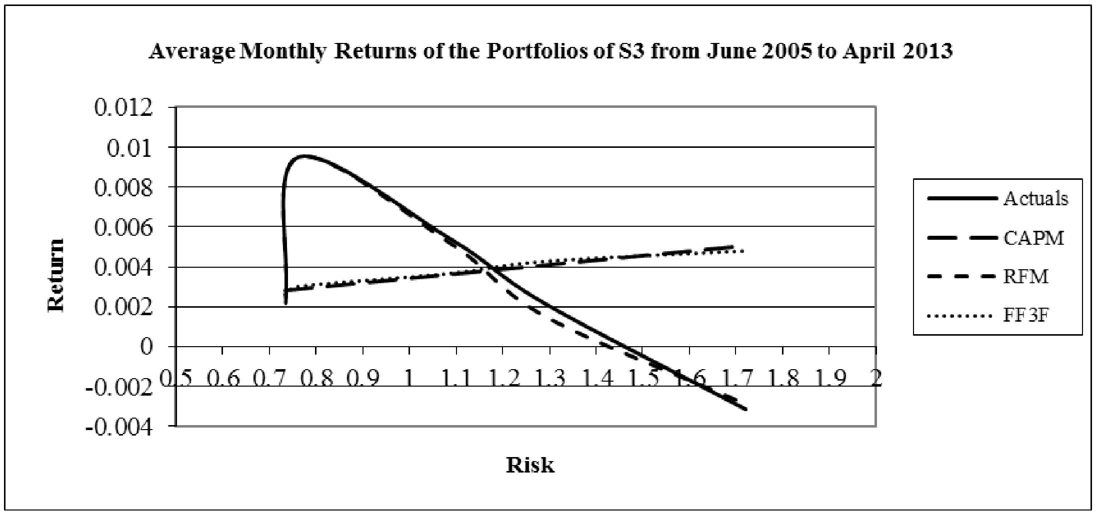
Portfolios	CAPM	FF3F	RFM	Improvement over CAPM	Improvement over FF3F
S1	9.93E-05	1.25E-04	4.07E-07	99.59%	99.68%
S2	9.72E-05	1.18E-04	4.33E-07	99.55%	99.63%
S3	1.19E-04	1.13E-04	5.90E-07	99.50%	99.48%
S4	1.07E-04	1.03E-04	6.00E-07	99.44%	99.42%
S5	3.12E-07	2.38E-07	1.81E-08	94.21%	92.40%
S6	4.85E-07	7.85E-07	2.40E-08	95.06%	96.95%
S7	7.13E-07	1.90E-07	8.04E-09	98.87%	95.78%
S8	8.52E-07	3.50E-07	7.89E-09	99.07%	97.74%
S9	7.12E-07	8.72E-07	1.05E-08	98.53%	98.80%
S10	3.04E-07	2.73E-07	6.95E-09	97.71%	97.45%
S11	8.80E-05	1.12E-04	3.79E-07	99.57%	99.66%

Figure 2: Charts of the Average Returns across Increasing Risk

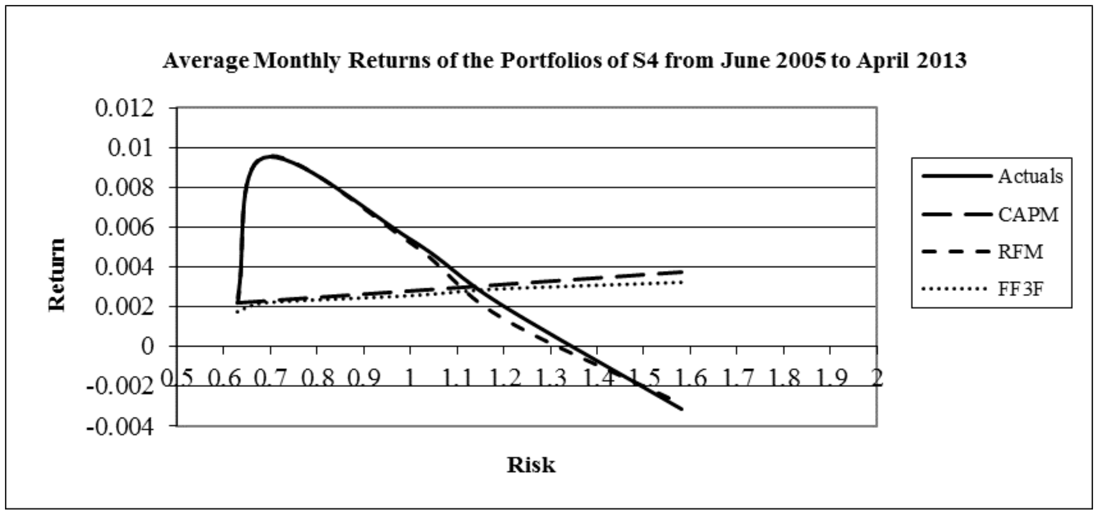




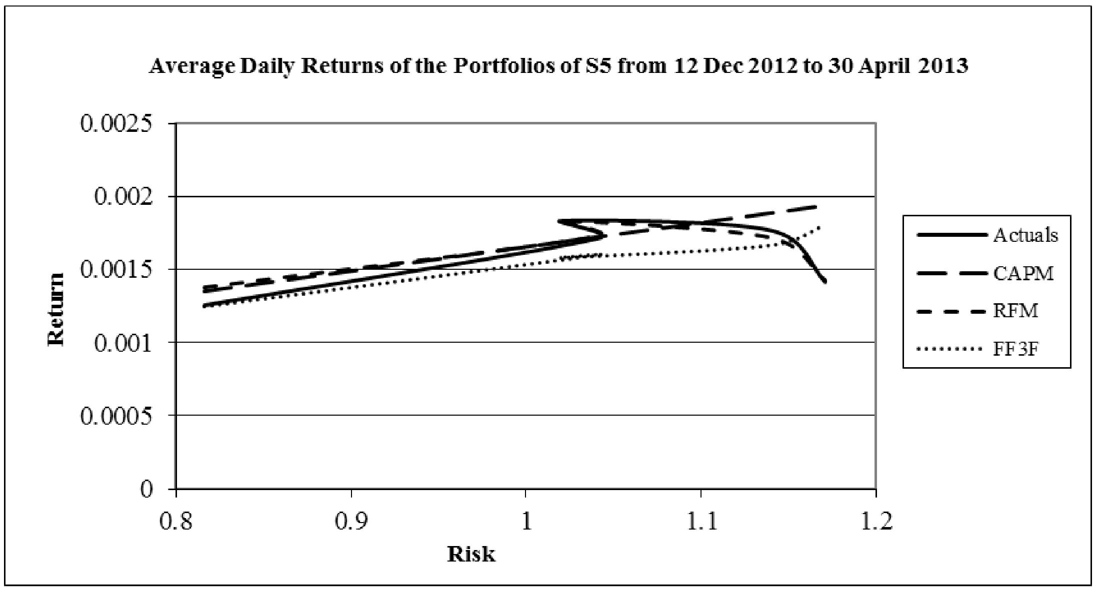
Average Monthly Returns of the Portfolios of S2 from May 2003 to April 2013
Return
0.01
0.008
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2
Risk
Actuals
CAPM
RFM
FF3F

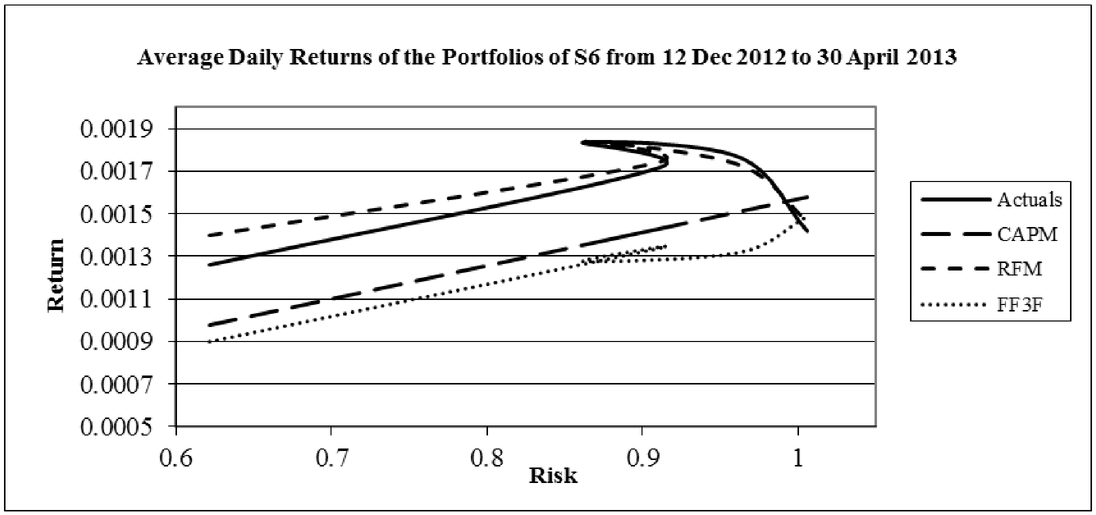


Average Monthly Returns of the Portfolios of S3 from June 2005 to April 2013
0.012
0.01
0.008
0.006
0.004
0.002
0
-0.002
-0.004
Return
0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2
Risk
Actuals
CAPM
RFM
FF3F

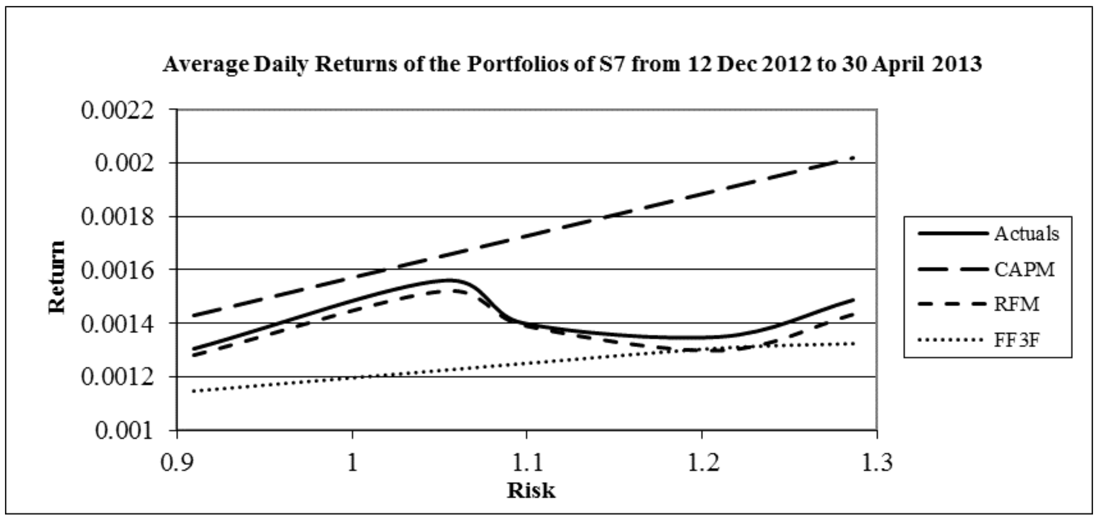


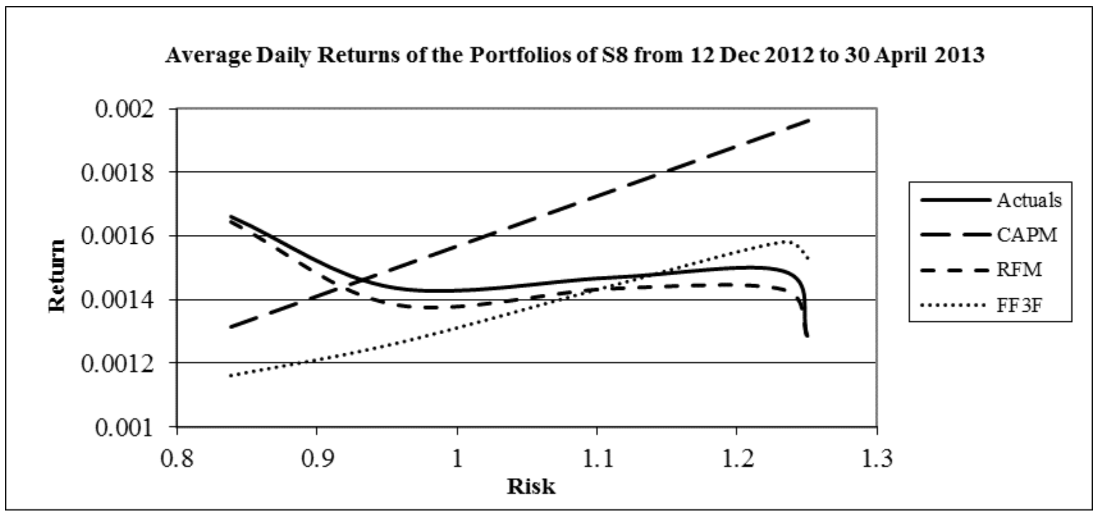
Average Monthly Returns of the Portfolios of S4 from June 2005 to April 2013
Return
0.012
0.01
0.008
0.006
0.004
0.002
0
-0.002
-0.004
0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2
Risk
Actuals
CAPM
RFM
FF3F

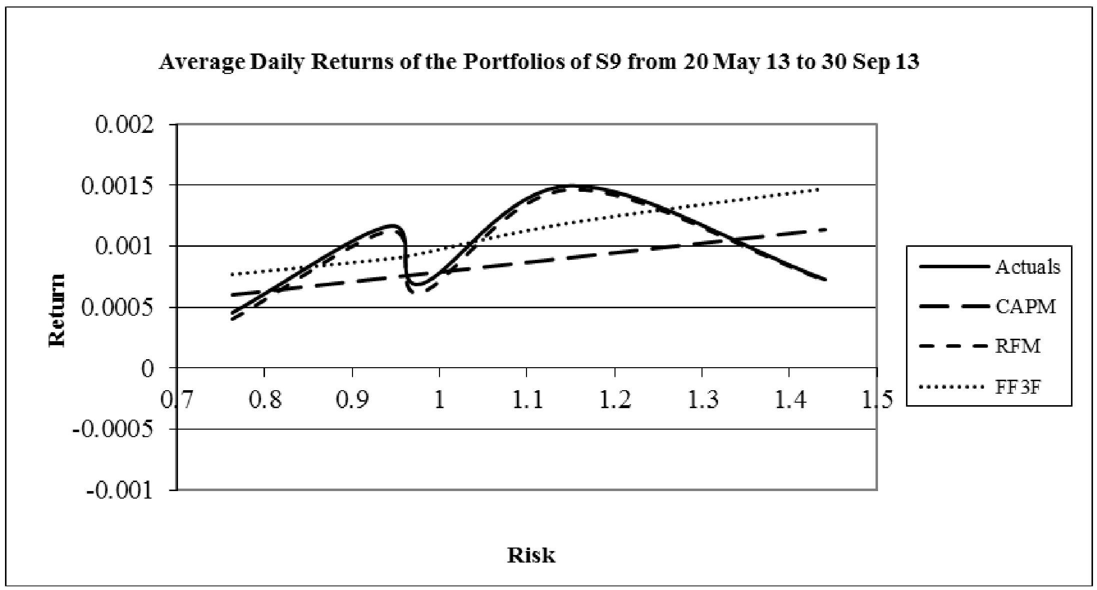

Average Daily Returns of the Portfolios of S5 from 12 Dec 2012 to 30 April 2013
0.0025
0.002
0.0015
0.001
0.0005
0
Return
0.8
0.9
1
1.1
1.2
Risk
Actuals
CAPM
RFM
FF3F



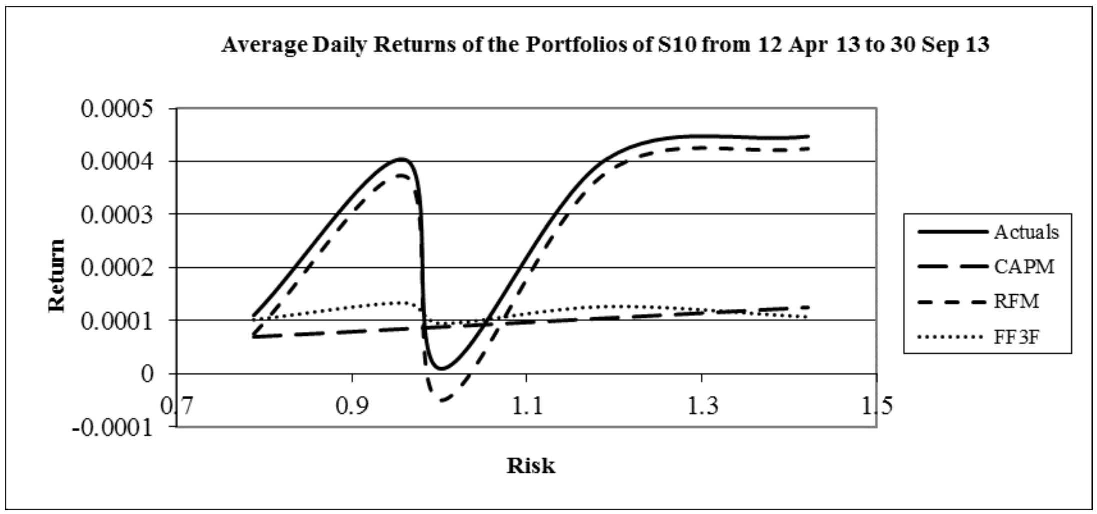
Average Daily Returns of the Portfolios of S6 from 12 Dec 2012 to 30 April 2013
Return
0.0019
0.0017
0.0015
0.0013
0.0011
0.0009
0.0007
0.0005
0.6
0.7
0.8
0.9
1
Risk
Actuals
CAPM
RFM
FF3F


Average Daily Returns of the Portfolios of S7 from 12 Dec 2012 to 30 April 2013
0.0022
0.002
0.0018
0.0016
0.0014
0.0012
0.001
Return
0.9
1
1.1
1.2
1.3
Risk
Actuals
CAPM
RFM
FF3F


Average Daily Returns of the Portfolios of S8 from 12 Dec 2012 to 30 April 2013
0.002
0.0018
0.0016
0.0014
0.0012
0.001
Return
0.8
0.9
1
1.1
1.2
1.3
Risk
Actuals
CAPM
RFM
FF3F



Average Daily Returns of the Portfolios of S9 from 20 May 13 to 30 Sep 13
0.002
0.0015
0.001
0.0005
0
-0.0005
-0.001
Return
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
Risk
Actuals
CAPM
RFM
FF3F


Average Daily Returns of the Portfolios of S10 from 12 Apr 13 to 30 Sep 13
0.0005
0.0004
0.0003
0.0002
0.0001
0
-0.0001
0.7
0.9
1.1
1.3
1.5
Return
Risk
Actuals
CAPM
RFM
FF3F

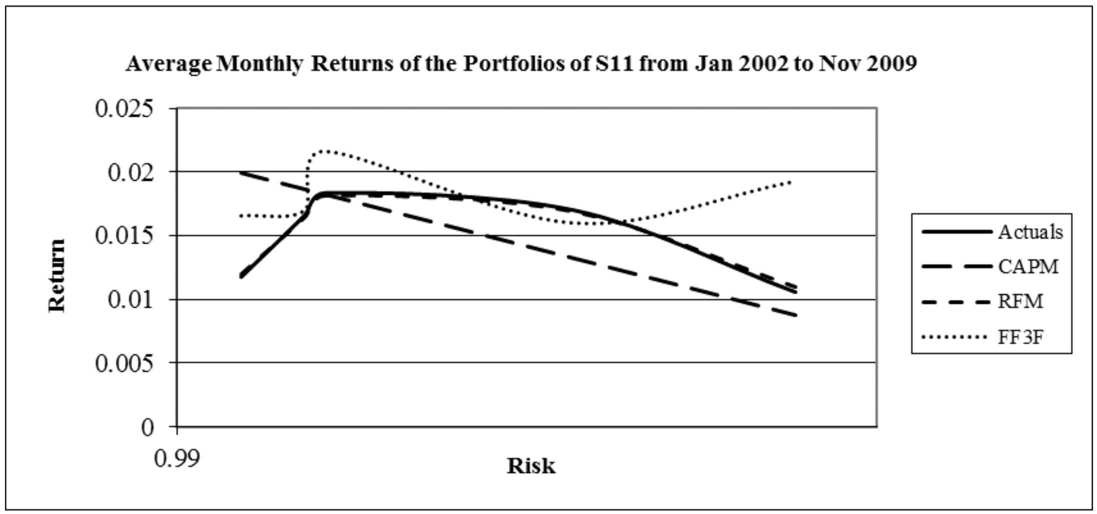

Average Monthly Returns of the Portfolios of S11 from Jan 2002 to Nov 2009
0.025
0.02
0.015
0.01
0.005
0
Return
0.99
Risk
Actuals
CAPM
RFM
FF3F

Table 8: Average Correlations between the Estimated Continuous Returns and the Actual Continuous Returns

Portfolios	CAPM	FF3F	RFM&FF3F
S1	89.85%	90.74%	91.03%
S2	88.57%	89.56%	90.02%
S3	90.93%	91.95%	92.42%
S4	89.54%	90.83%	91.38%
S5	85.73%	87.07%	87.95%
S6	84.90%	86.21%	86.91%
S7	92.27%	96.32%	96.57%
S8	95.89%	97.01%	97.17%
S9	92.03%	92.74%	93.03%
S10	90.29%	91.88%	91.97%
S11	92.61%	93.01%	93.24%

Table 9: Sum of Squared Errors (SSE) between the Estimated and the Actual 'Continuous' Returns

Portfolios	SSE_{CAPM}	SSE_{FF3F}	$SSE_{RFM\&FF3F}$	$(SSE_{CAPM} - SSE_{FF3F})/SSE_{CAPM}$	$(SSE_{CAPM} - SSE_{RFM\&F3F})/SSE_{CAPM}$	$(SSE_{FF3F} - SSE_{RFM\&F3F})/SSE_{FF3F}$
S1	5.66E-02	5.00E-02	4.75E-02	11.58%	16.10%	5.11%
S2	5.87E-02	5.30E-02	5.01E-02	9.64%	14.67%	5.56%
S3	4.88E-02	4.02E-02	3.72E-02	17.76%	23.76%	7.29%
S4	5.18E-02	4.31E-02	3.98E-02	16.82%	23.13%	7.59%
S5	1.23E-03	1.12E-03	1.06E-03	9.34%	14.01%	5.15%
S6	1.26E-03	1.16E-03	1.12E-03	7.98%	11.59%	3.92%
S7	1.01E-03	4.84E-04	4.44E-04	52.02%	55.99%	8.28%
S8	4.69E-04	3.21E-04	2.95E-04	31.47%	37.12%	8.25%
S9	1.46E-03	1.34E-03	1.29E-03	8.67%	12.15%	3.81%
S10	2.37E-03	1.99E-03	1.97E-03	15.98%	17.04%	1.27%
S11	1.18E-01	9.80E-02	9.45E-02	16.88%	19.88%	3.61%
			Paired t-test	H_0: $(SSE_{CAPM} - SSE_{FF3F})/SSE_{CAPM} \leq 0$	H_0: $(SSE_{CAPM} - SSE_{RFM\&FF3F})/SSE_{CAPM} \leq 0$	H_0: $(SSE_{FF3F} - SSE_{RFM\&FF3F})/SSE_{FF3F} \leq 0$
			t-statistic	3.07	3.75	5.44
			pvalue	1.39%	0.67%	0.14%