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Options Pricing with Skewness and Kurtosis Adjustments

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Abstract

The pricing of options is one of the most complex areas of applied finance and has been a subject of extensive study. Understanding the intricacies of this pricing and the trends therein is necessary for an investor who wants to trade in options. The Black Scholes Model is considered as an elegant piece of research into option prices. Subsequently, many models have been developed, some of which are largely extensions and modifications to the Black – Scholes model. The efficiency of these models to predict the option prices to the most accurate level or to the level of minimum deviation has been a subject for various empirical studies. While Black Scholes model is considered to be a big success in financial theory both in terms of approach and applicability, the model suffers from various deficiencies. This paper is aimed at applying the corrections suggested by Corrado-Su to the Black Scholes Model using Gram-Charlier (CG) expression for option pricing in Indian market. The study applies the Corrado-Su formula to price the options on equity as well as index options. The stocks which make up more than 60 % weights of the NIFTY Index have been considered for applying the Corrado-Su correction and the results of pricing efficiency are compared with the same for index option. Data pertaining to July-Sept 2013 are used in the study. Based on the results, it can be concluded that the Corrado-Su modified B-S formula provides a viable alternative to the B-S model by reducing the deviations and thereby improving the pricing approximation.

1. Introduction

In today's financial world there is a great need to predict the value of assets, using which strategic decisions can be made to make short term or long term capital gains. Due to the dynamic and uncertain nature of the financial markets, the prediction of the asset prices is really difficult. Many models have been developed to predict the option prices in the financial market. The efficacy of these models to predict the option prices to the most accurate level or to the level of minimum deviation has been tested in various markets.

2. Literature Review

Black Scholes model is considered the biggest success in the financial theory both in terms of approach and applicability. The strength of the Black-Scholes model (1973) is the possibility of estimating market volatility of an underlying asset generally as a function of price and time. Its second strong point is the self-replicating strategy or hedging i.e. an explicit trading strategy in underlying assets and risk-less bonds whose terminal payoff is equal to payoff of a derivative security at maturity. Despite its usefulness the model has various deficiencies.

Macbeth and Merville (1979) found that out-of-the-money call options were overpriced by BS model and in-the-money call options were under-priced by BS model. These effects became more pronounced as the time to maturity increased and the degree to which the option is in or out of the money increased.

Rubinstein (1985) derives a relatively simple method to extend the Black-Scholes formula to account for non-normal skewness and kurtosis in stock return distributions. One of the deficiencies of Black-Scholes model includes frequently mispricing deep-in-the-money and deep-out-of-the-money options. Rubinstein reports a mispricing pattern, where the Black-Scholes model under-prices out-of-the-money options and overprices in-the-money options.

In all major markets around the world, different implied volatilities of options on the same underlying asset across different exercise prices and terms to maturity have been observed. In a study on the NSE NIFTY, Misra, Kannan and Misra (2006) have reported a significant volatility smile on NIFTY options. The results of their study show that deep-in-the-money and deep-out-of-the-money options have higher volatility than at-the-money options and that the implied volatility of out-of-the-money call options is greater than in-the-money calls. Daily returns of the NSE NIFTY have been found to follow normal distribution with some Skewness and Kurtosis. These results suggest that the volatility smile observed in the NSE NIFTY options can be explained in some measure by the observed Skewness and Kurtosis.

Tripathi & Gupta (2010) tested the predictive accuracy of the Black-Scholes (BS) model in pricing the Nifty Index option contracts by examining whether the skewness and kurtosis adjusted BS model of Corrado and Su gives better results than the original BS model. It was also examined whether volatility smile in case of NSE Nifty options, if any, can be attributed to the non-normal skewness and kurtosis of stock returns. Based on data of S&P CNX NIFTY near-the-month call options for the period January 1, 2003 to December 24, 2008, their results show that BS model is misspecified as the implied volatility graph depicts the shape of a 'Smile' for the study period. There is significant under-pricing by the original BS model and that the mispricing increases as the moneyness increases. Even the modified BS model misprices options significantly. However, pricing errors are less in case of the modified BS model than in case of the original BS model. On the basis of Mean Absolute Error (MAE), they concluded that the modified BS model is performing better than the original BS model.

3. The Black Scholes Model

The Black Scholes formula consists of constantly changing factors, the hedge portfolio comprising a long position in the stock and a short position in the zero-coupon bond. The hedge portfolio will be constituted in such a way that at any given point of time its value will always be equal to the option's price at that time. So, the portfolio is called as dynamic portfolio and the act of maintaining the portfolio in balance is called as hedge rebalancing.

$c = S_0 N(d_1) - K e^{-rT} N(d_2)$, where

$$d_1 = \frac{[\ln(S_0/K)+(r+\sigma^2/2)T]}{\sigma\sqrt{T}}$$

$$d_2 = \frac{[\ln(S_0/K)+(r-\sigma^2/2)T]}{\sigma\sqrt{T}}$$

The variable c is the European Call price, S_0 is the stock price at time zero, K is the strike price, r is the continuously compounded risk free rate, σ is the stock price volatility and T is the time to maturity of the option. The function N(x) is the cumulative probability density function for a standard normal distribution. In other words, it is the probability that the variable with a standard normal distribution Z (0, 1) will be less than x.

The model is based on certain assumptions which may not be possible to realize in real terms. These assumptions are stated below:

a) Volatility, σ - a measure of how much a stock can be expected to move in the near term - is constant over time.

b) Returns on the stock prices are normally distributed.

c) The risk-free interest rate, r, is constant and the same for all maturities.

d) Security trading is continuous.

e) Markets are perfectly liquid and it is possible to purchase or sell any amount of stock or options or their fractions at any given time.

f) The underlying stock does not pay dividends during the option's life.

g) The model assumes European-style options which can only be exercised on the expiration date.

Although the assumptions under d) and e) are realised in ideal markets, we are assuming an efficient and complete market in any case. While the model can be tweaked approximately for an American option, the Assumptions under f) dividend pay-out, a) volatility and b) log normal distributions are discussed below.

Merton (1973) suggested a modified formula to account for the dividends. A common way of adjusting the Black-Scholes model for dividends is to subtract the discounted value of a future dividend from the stock price. This modification also provides the option pricing formula for index options.

B- S Formula for Index Option Pricing

$$c = S_0 e^{-qT} N(d_1) - K e^{-rT} N(d_2)$$

where $d_1 = \frac{[\ln(S_0/K) + (r - q + \sigma^2/2)T]}{\sigma\sqrt{T}}$

$d_2 = \frac{[\ln(S_0/K) + (r - q - \sigma^2/2)T]}{\sigma\sqrt{T}}$ and

q is the dividend yield of the Index.

The assumption of constant volatility is naturally replaced by historical volatility and especially by implied volatility. The implied volatility is the value of statistical volatility needed to be used in the standard Black-Scholes pricing formula for a given day to yield the market prices of that option for the day under different moneyness and expiry..

Another assumption of the Black Scholes (B-S) Equation is that of normality of stock returns. However actual data of stock returns has been found to be non-normal in many markets. Also, while it has been observed that the implied volatilities for different days and expiry of options form a pattern of either "smile" or "skew", Hull (2010) attributes the volatility smile to the non-normal Skewness and Kurtosis of stock returns.

4. Skewness & Kurtosis adjusted Black-Scholes Model

Researchers around the world have documented strike price bias and a time-to-maturity bias in Black-Scholes Model. If these biases were caused by the violation of the assumption that the terminal asset price is log-normally distributed, one method of correcting the bias would be to assume a different terminal asset price distribution, one that more closely approximates the true underlying distribution.

Jarrow and Rudd (1982) proposed a semi parametric option pricing model to account for observed strike price biases in the Black-Scholes model. They derive an option pricing formula from an expansion of the lognormal probability density function to model the distribution of stock prices. Jarrow & Rudd first used an Edgeworth expansion of the log-normal density function to write the option price as a function of the third and fourth movements of the terminal price distribution. The first two movements of the approximating distribution remain the same as that of the normal distribution, but third and fourth moments are introduced as the higher order terms of expansion. Operationally Jarrow and Rudd method accounts for skewness and kurtosis deviations from log normality of stock prices.

Corrado and Su (1996) have extended the Black-Scholes formula to account for non-normal skewness and kurtosis in stock return distributions. Their assumption is that if the volatility smile is due to non-normal skewness and kurtosis of the distribution of asset returns, this would be removed if the effect of this deviation is included in the pricing formula. The method developed by Corrado and Su accounts for skewness and kurtosis deviations from normality of stock returns. The skewness and kurtosis coefficients are simultaneously estimated with an implied standard deviation. Their method accounts for biases induced by non-normal skewness and kurtosis in stock return distributions and adapt a Gram-Charlier series expansion of the normal density function to provide skewness and kurtosis adjustment terms for the Black-Scholes formula.

To incorporate option price adjustments for non-normal skewness and kurtosis in an expanded Black-Scholes option pricing formula, Corrado & Su (1996) used a Gram-Charlier series expansion of a normal density function. The following option price formulas are obtained based on a Gram-Charlier density expansion, denoted here by $\mathbf{C_{GC}}$:

$$\mathbf{C_{GC} = C_{BS} + \mu_3{*}Q_3 + (\mu_4 - 3){*}Q_4}$$

Where C_{BS} is the Black-Scholes option pricing formula and Q_3&Q_4 represent the marginal effect of non-normal skewness (μ_3) & kurtosis (μ_4) respectively.

Skewness & Kurtosis Adjusted Formula for Equity Option Pricing:

$C_{GC} = C_{BS} + \mu_3 * Q_3 + (\mu_4 - 3) * Q_4$, where

$$C_{BS} = S_0\, N(d) - K\, e^{-rT} N(d - \sigma\sqrt{T})$$

$$Q_3 = \frac{1}{3!} S_0 \sigma\sqrt{T}\left(\left(2\,\sigma\sqrt{T} - d\right)N(d) + \sigma^2 T N(d)\right)$$

$$Q_4 = \frac{1}{4!} S_0 \sigma\sqrt{T}\left(\left(d^2 - 1 - 3\,\sigma\sqrt{T}(d - \sigma\sqrt{T})\right)N(d) + \sigma^3 T^{3/2} N(d)\right)$$

Skewness & Kurtosis Adjusted Formula for Index Option Pricing:

$C_{GC} = C_{BS} + \mu_3 * Q_3 + (\mu_4 - 3) * Q_4$, where

$C_{BS} = S_0\, e^{-qT} N(d) - K\, e^{-rT} N(d - \sigma\sqrt{T})$, where

$$Q_3 = \frac{1}{3!} S_0 e^{-qT} \sigma\sqrt{T}\left(\left(2\,\sigma\sqrt{T} - d\right)N(d) + \sigma^2 T N(d)\right)$$

$$Q_4 = \frac{1}{4!} S_0 e^{-qT} \sigma\sqrt{T}\left(\left(d^2 - 1 - 3\,\sigma\sqrt{T}(d - \sigma\sqrt{T})\right)N(d) + \sigma^3 T^{3/2} N(d)\right)$$

where q is the dividend yield of the Index.

Their first set of estimation procedures assesses the performance of the Black-Scholes option pricing model. They estimated Implied Volatility (IV) on a daily basis for call options on the S&P 500 Index using Whaley's (1982) simultaneous equations procedure. This IV is used as an input to calculate theoretical Black-Scholes option price for all price observations within the same maturity class. These theoretical Black-Scholes prices were then compared with corresponding market-observed prices.

The second set of estimation procedures assesses the performance of the skewness and kurtosis adjusted Black-Scholes option pricing formula discussed above. In these procedures simultaneous estimation of IV, Implied Skewness (ISK) and Implied Kurtosis (IKT) parameters on a given day for a given maturity class was done. These theoretical skewness and kurtosis adjusted Black-Scholes option prices were then compared with corresponding market-observed prices.

While the Corrado-Su formula emanated followed the Jarrow- Rudd formula, Jarrow and Rudd (1982) derived an option pricing formula from an expansion of the lognormal probability density function to model the distribution of stock prices. Operationally Jarrow and Rudd method accounts for skewness and kurtosis deviations from log normality of stock prices, while the method developed by Corrado and Su accounts for skewness and kurtosis deviations from normality of stock returns. In contrast, skewness and kurtosis coefficients, which are 0 & 3 respectively for all the normal distributions, vary across different lognormal distributions. According to Corrado and Su, it is more convenient to report and interpret empirical result based on observed skewness and kurtosis deviations from a normal distribution.

Brown and Robinson (2002) provide a typographic correction to the expression for the skewness coefficient derived by Corrado & Su. They also proved that the size of the absolute error in pricing using the incorrect formula varies with the moneyness of the option.

The Brown & Robinson Correction:

The standard definition of the Hermite polynomial (Stuart & Ord 1994) is

$$H_n(z)n(z) = (-1)^n \frac{d^n n(z)}{dz^n}$$

Corrado & Su (1996) defined the Hermite Polynomial as:

$$H_n(z)n(z) = \frac{d^n n(z)}{dz^n}$$

The expression of Q3 must then be altered from

$$Q_3 = \frac{1}{3!} S_0 \sigma \sqrt{t}[(2\sigma\sqrt{t} - d)n(d) - \sigma^2 t N(d)]$$

to this corrected expression

$$Q_3 = \frac{1}{3!} S_0 \sigma \sqrt{t}[(2\sigma\sqrt{t} - d)n(d) + \sigma^2 t N(d)]$$

Then, using this result, call option price is given by:

$C_{GC} = C_{BS} + \mu_3 * Q_3 + (\mu_4 - 3) * Q_4$

The correction is incorporated appropriately in the formulas given above.

5. Calculation of Implied Volatility

The four parameters of Black Scholes option pricing formula, viz; Stock Price, Strike Price and Time to Maturity of the option are directly observed from the market. Another input to the formula is the volatility (Standard Deviation) of stock price which cannot be observed. This should theoretically be identical for options of all strike prices because the underlying asset is the same in each case. But, since this is not directly observable, it has been estimated using the following method. Using option prices for all contracts within a given maturity series observed during a quarter, we estimate a single implied volatility to minimize the total error sum of squares between the predicted and the market prices of options of various strike prices. This has been calculated using Microsoft Excel Solver function by minimizing the following function:

$$\frac{min}{\sigma} \sum_{j=1}^{N} \left[C_{obs} - C_{BS,j}\right]^2$$

In the above equation, N is the total number of price quotations available on a given day for a given maturity class, C_{obs} is the market observed call price, and C_{BS} is theoretical Black Scholes call price calculated using the implied volatility (σ) as the parameter. The minimization of the above equation is achieved using solver in the Microsoft Excel.

The implied volatility for call option is calculated on the basis of Contract Date, maturity and Strike Price. On a given day for given option maturity class, a unique implied volatility from all options is obtained using Whaley's (1982) procedure. The equations are solved in Microsoft Excel using solver. This unique implied volatility is used as an input to calculate the B-S option prices for all price observations within the same maturity class for the next set of data (next contract date). These prices (IVW-OOS) are then compared with the corresponding market observed prices.

Next, we assess the skewness and kurtosis adjusted Black Scholes option pricing formula developed by Corrado & Su (1997). Specifically, during a day, we estimate a single implied volatility, a single skewness coefficient, and a single excess kurtosis coefficient by minimizing once again the error sum of squares represented by the following formula.

$$\frac{min}{\sigma, \mu_3, \mu_4} \sum_{j=1}^{N} \left[C_{obs,j} - (C_{BS,j}(\sigma) + \mu_3 Q_3 + (\mu_4 - 3)Q_4)\right]^2$$

Where σ, μ_3& μ_4 represent estimates of the implied volatility, implied skewness and implied kurtosis parameters based on N price observations. We then use the three parameter estimates as inputs to the Corrado & Su formula to calculate theoretical option prices corresponding to all option prices within the same maturity series observed during the following day.

6. Research Methodology & Data Collection

The following empirical study was carried out on 12 stocks for a span of 3 months ranging from 1st July, 2013 to 30th Sep, 2013. These 12 stocks constitute the NIFTY Index making up at least 60% of the average weightage of the NIFTY Index.

The data was collated and sorted on the basis of Expiry date, Contract Date and then Strike Price of option. The following calculations were performed in a series of stages:

1) Calculate theoretical call option price using implied volatility calculated using Whaley's procedure in Black Scholes formula (BSIVW -OOS).

2) Calculate theoretical call option price using adjusted B-S option pricing formula suggested by Corrado & Su (C_{GC} - OOS) after inputting the values of skewness and kurtosis parameters and their coefficients.

3) Calculate the squared differences and compare the above stages through the mean sum of squares (MSE).

4) Conduct the Student's t-test and Wilcoxon test for medians for the out-of-sample data i.e. Cgc –oos and test the statistical significance.

7. Sample Selection

The main source of data collection was secondary from NSE website. The sample data was constituted of stock options of 12 companies which constituted at least more than 60% weightage of the total NIFTY Index (at the beginning of Sep, 2013). The data of NIFTY Index was also collected for the comparison of results. The details are given in table 1 below.

Table 1: Details of weightages of different equities in the NIFTY index

Security Symbol	**Security Name**	**Weightage (%) in NIFTY index**			
		July-13	**Aug-13**	**Sep-13**	**Average**
ITC	I T C Ltd.	10.26	9.73	10.02	10.00
INFY	Infosys Ltd.	7.83	8.59	7.78	8.07
RELIANCE	Reliance Industries Ltd.	7.83	8.05	7.23	7.70
HDFC	Housing Development Finance Corporation Ltd.	6.81	6.41	6.37	6.53
HDFCBANK	HDFC Bank Ltd.	6.15	6.29	5.86	6.10

ICICIBANK	ICICI Bank Ltd.	5.74	5.32	5.46	5.51
TCS	Tata Consultancy Services Ltd.	5.06	5.95	5.26	5.42
LT	Larsen & Toubro Ltd.	3.77	3.38	3.44	3.53
TATAMOTORS	Tata Motors Ltd.	2.84	3.08	3.20	3.04
ONGC	Oil & Natural Gas Corporation Ltd.	2.81	2.53	2.54	2.63
SUNPHARMA	Sun Pharmaceutical Industries Ltd.	2.33	2.25	2.39	2.32
HINDUNILVR	Hindustan Unilever Ltd.	2.37	2.56	2.38	2.44
		63.8	**64.14**	**61.93**	**63.29**

8. Data Collection

The data for the call option of the stated 12 stocks for the period from 1st July, 2013 to 30th Sep, 2013 was collected from the NSE website and collated on the basis of contract date.

The data was further filtered to eliminate option contracts which were thinly traded. And the elimination criteria included the options with zero settlement prices, options with number of contracts less than or equal to 5 for a particular strike price on a single day, options with Time to expiry less than 5 days. The data was then sorted based upon Expiry date, Contract Date and then strike price. Thus the data obtained from the NSE website was sorted into 15 different categories according to the Time to Expiry (1M, 2M, and 3M) and moneyness of the options. The moneyness is identified on the basis of variation of Strike Price of option with its underlying price of stock. The extreme outliers of this variation have been filtered out to eliminate skewness of results and the resultant categorisation is shown in table 2.

Table 2: Categorisation of moneyness

	Underlying Price (S) / Strike Price (K)	**Deep Moneyness category**
1	S/K > 1.15	Filtered Out
2	1.10 < S/K > 1.15	DITM
3	1.03 < S/K < 1.10	ITM
4	0.97 < S/K < 1.03	ATM
5	0.90 < S/K < 0.97	OTM
6	0.85 < S/K < 0.90	DOTM
7	S/K < 0.85	Filtered Out

9. Risk Free Interest Rate

In developed markets, risk-free rate of interest is calculated by the yield of treasury bills which matures on the same date of expiration of the options. Since in India the T-Bill market is not mature and deep, NSE itself uses MIBOR (Mumbai Inter Bank Offer rate) as the risk free rate of interest. The MIBOR rate was downloaded from the NSE website and used as the risk free rate.

10. Time to Expiry

When Time to Expiry is used in the formula as 'e^{-rt}', 't' is the time left for options to expire. In India, interest is calculated by banks and other financial inter-intermediary based on calendar days, irrespective of the number of intervening holidays during the period. Time to Expiry is annualized by dividing the number of days left for the option to expire by the total number of calendar days (i.e. 365 days) in a year.

11. Test Results and Discussion

Testing for Normality

The Kolmogorov- Smirnov test assesses whether there is a significant departure from normality in the population distribution for the different stock prices. The one sample K-S test takes the observed cumulative distribution of the data and compares them to the theoretical cumulative distribution for a normally distributed population.

Our assumption about the prices is actually log-normality and we need to test for log-normality. If the returns are log-normally distributed, the log of the prices is normally distributed. For the same, we create a new variable of logprice (through SPSS –compute new variable) and run the test. The details of test results are given in table 3.

Hypothesis for Normality:

H_0: The distribution of Natural Logarithm of Underlying Price is Normal.

H_1: The distribution of Natural Logarithm of Underlying Price is Non-Normal.

Table 3: Summary of K-S Test of Normality for Equity & Index Options

	Logprice distribution		K-S Test	Hypothesis
	MEAN	SD	Significance value	
HDFC	6.67	.06	0.00	null rejected
HDFCBANK	6.44	.06	0.018	null rejected
HINDUNILVR	6.44	.05	0.00	null rejected
ICICIBANK	6.83	.09	0.00	null rejected
INFY	7.97	.07	0.00	null rejected
ITC	5.82	.06	0.001	null rejected
LT	6.78	.20	0.00	null rejected
ONGC	5.65	.07	0.057 (marginal)	**null retained**

RELIANCE	6.76	.04	0.001	null rejected
SUNPHARMA	6.52	.32	0.00	null rejected
TATAMOTORS	5.73	.07	0.00	null rejected
TCS	7.50	.09	0.00	null rejected
NIFTY	8.66	.04	0.00	null rejected

The p-values range from 0.00 to 0.057 (ONGC). The case of null retention for ONGC is at the margin of 5% rejection and can be discounted.

Hence we reject the null hypothesis of normality and conclude that the logprices are not normal in all the samples. Consequently the stock and index prices are not lognormal.

Testing for Differences

Over the past few decades after the introduction of B-S formula, research has been conducted on time-to-maturity and strike-price biases. If these biases are leading to the violation of the assumption of the lognormal distribution of the terminal prices, we can think about correcting the bias. Our concern takes the form of assuming a different terminal stock price distribution, which is more closely approximating the true underlying distribution.

For further hypothesis testing, however, standard procedures of testing of option pricing models are followed and t-tests were performed assuming log- normality of stock prices. The results are interpreted from the test results of nonparametric Wilcoxon test, having no distributional assumptions. The results of t-tests are used as additional quantitative interpretation of the non-parametric results.

Related Sample Wilcoxon Signed Rank test with 95 % Confidence Level

Hypothesis for CGC-OOS:

H_0: There is no significant difference between the medians of CGC-OOS call prices and Market call prices.

H_2: There is a significant difference between the medians of CGC-OOS call prices and Market call prices.

12. Discussion of Results

Samples of 12 equity options (Period Jul-Sep, 2013)

The summary of Wilcoxon test results are given in table 4.

It can be seen from table 5 that there is definite improvement from IVW-OOS to CGC-OOS, indicating that the Corrado-Su formula works better than the original B-S formula with implied volatility.

It is also worthwhile to further analyse the cases of null retained further to develop a better understanding of the issue at hand. This has been done through an analysis of categories as well as percentages of observations in null retained categories. The details are presented in table 6.

Table 4: Summary of Results from Wilcoxon Non-parametric Test for Equity Options

	ATM		**DITM**		**DOTM**		**ITM**		**OTM**	
	1M	**2M**	**1M**	**2M**	**1M**	**2M**	**1M**	**2M**	**1M**	**2M**
HDFC	Retain	Retain	Retain	-	Retain	Retain	Retain	Retain	Retain	Retain
HDFCBANK	Retain	Retain	-	-	Retain	Retain	Retain	-	Retain	Retain
HINDUNILVR	Retain	Retain	Retain	Retain	Retain	Retain	Retain	Retain	Retain	Retain
ICICBANK	Retain	Retain	Retain	-	Retain	Retain	Retain	Retain	Retain	Retain
INFY	Retain	Reject	Retain	Reject	Reject	Reject	Retain	Retain	Reject	Reject
ITC	Retain	Retain	Retain	Retain	Reject	Retain	Retain	Retain	Retain	Retain
LT	Reject	Retain	Retain	-	Reject	Retain	Retain	Retain	Reject	Retain
ONGC	Retain	Retain	Reject	-	Retain	Retain	Retain	Retain	Retain	Retain
RELIANCE	Retain	Retain	Retain	Retain	Reject	Retain	Retain	Retain	Retain	Retain
SUNPHARMA	Retain	Retain	-	-	Retain	-	Retain	Retain	Retain	Retain
TATAMOTORS	Retain	Retain	Retain	Retain	Reject	Retain	Retain	Retain	Retain	Retain
TCS	Reject	Retain	Retain	-	Reject	Retain	Retain	Retain	Reject	Retain

Table 5: Analysis of Null Retained –Equity Options- Combinations

	IVW-OOS			**CGC-OOS**		
EQUITY	**Null Retained**	**Null Rejected**	**Total**	**Null Retained**	**Null Rejected**	**Total**
Moneyness						
ATM	18	6	24	21	3	24
DITM	13	2	15	13	2	15
DOTM	11	12	23	16	7	23
ITM	18	5	23	23	0	23
OTM	14	10	24	20	4	24
Expiry						
1M	36	22	58	46	12	58
2M	38	13	51	47	4	51

An analysis of Moneyness Categories reveals that the formula fares medium in the case of DOTM (57%) whereas in other categories, the percentages range from 72 to 100 %. The overall percentage for all equity options put together comes to 75%.

An analysis of time-to - expiry categories reveals that the percentages are 71 % and 89 % for 1-month and 2-months respectively. There were no samples for 3-month expiry in the case of equity options.

Table 6: Analysis of Null Retained-Equity Options - Percentages

Equity	**Null Retained**	**Null Rejected**	**Total obs**	**% Retention**
Overall	5520	1790	7310	75.51
Moneyness				
ATM	1778	521	2299	77.34
DITM	133	13	146	91.10
DOTM	616	466	1082	56.93
ITM	938	0	938	100.00
OTM	2055	790	2845	72.23
Expiry				
1M	3958	1615	5573	71.02
2M	1562	175	1737	89.93

Based on the results, it can be inferred that the Corrado-Su modified B-S model (CGC-OOS) is quite efficient in producing theoretical call prices with much less deviations than the original model with implied volatility (IVW-OOS).

Sample of NIFTY index options (Period Jul-Sep, 2013)

The summary of Wilcoxon test results are given in table 7.

Table 7: Summary of Results from Wilcoxon Non-parametric Test for NIFTY index options

NIFTY Index	ATM	DITM	DOTM	ITM	OTM
1M	Retain	Reject	Reject	Retain	Reject
2M	Reject	Retain	Reject	Reject	Retain
3M	Reject	Retain	Reject	Retain	Reject

Table 8: Analysis of Null Retained – NIFTY Options - Combinations

	IVW-OOS			**CGC-OOS**		
NIFTY	**Null Retained**	**Null Rejected**	**Total**	**Null Retained**	**Null Rejected**	**Total**
Moneyness						
ATM	0	3	3	1	2	3
DITM	3	0	3	2	1	3
DOTM	0	3	3	0	3	3
ITM	0	3	3	2	1	3
OTM	0	3	3	1	2	3
Expiry						
1M	1	4	5	2	3	5
2M	1	4	5	2	3	5
3M	1	4	5	2	3	5

It can be seen from the table 8 that there is some improvement from IVW-OOS to CGC-OOS, indicating that the Corrado-Su formula works better than the original B-S formula with implied volatility.

It is also worthwhile to further analyse the cases of null retained further to develop a better understanding of the issue at hand. This has been done through an analysis of categories as well as percentages of observations in null retained categories and details are presented in table 9.

Table 9: Analysis of Null Retained – NIFTY Options - Percentages

Index	Null Retained	Null Rejected	Total obs	% Retention
Overall	1031	2244	3275	31.48
Moneyness				
ATM	218	552	770	28.31
DITM	138	89	227	60.79
DOTM	0	608	608	0.00
ITM	349	207	556	62.77
OTM	326	788	1114	29.26
Expiry				
1M	407	524	931	43.72
2M	431	637	1068	40.36
3M	193	1083	1276	15.13

An analysis of Moneyness Categories reveals that the formula fares poorly in the case of DOTM (0%) whereas in other categories, the percentages range from 28 to 63%. The overall percentage for all equity options put together comes to 32%.

An analysis of time-to - expiry categories reveals that the percentages are 44, 40 and 15% for 1-month, 2-months, and 3-months respectively.

The results for NIFTY Index are drastically different from that of Equity Stocks wherein H0, the Null Hypothesis of no difference is rejected in 9 out of 15 combinations, i.e. there is a significant difference in the calculated options price (CGC-OOS) and the observed Call Market price of options.

From the results, it can be inferred that modified B-S Model is not able to produce efficient results for NIFTY index option in case of At-the-money, Out-of-the Money and Deep Out-of-the-Money options. The same formula is able to produce better results for In-the-Money and Deep-In-the-Money options.

13. Conclusions

Based upon the foregoing discussions, we conclude that:

1) There is a definite improvement with the Corrado-Su modified formula for equity option pricing, as compared to the original B-S formula with implied volatility.

2) The improvement in the case of NIFTY index option pricing is not much and also not significant.

3) The Corrado-Su adjustments seem to work well with equity options but not with index options.

4) We are unable, at this stage, to explain the failure of the Corrado-Su formula in the case of NIFTY index options. This may need further research.

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